Chapter 12: Indexing and Hashing (Cnt.)

- Basic Concepts
- Ordered Indices
- B+-Tree Index Files
- B-Tree Index Files
- Static Hashing
- Dynamic Hashing
- Comparison of Ordered Indexing and Hashing
- Index Definition in SQL
- Multiple-Key Access
Dynamic Hashing

- Good for database that grows and shrinks in size
- Allows the hash function to be modified dynamically

**Extendable hashing** – one form of dynamic hashing

- Hash function generates values over a large range — typically $b$-bit integers, with $b = 32$.
- At any time use only a prefix of the hash function to index into a table of bucket addresses.
- Let the length of the prefix be $i$ bits, $0 \leq i \leq 32$.
- Bucket address table size = $2^i$. Initially $i = 0$
- Value of $i$ grows and shrinks as the size of the database grows and shrinks.
- Multiple entries in the bucket address table may point to a bucket.
- Thus, actual number of buckets is $< 2^i$
  - The number of buckets also changes dynamically due to coalescing and splitting of buckets.
In this structure, $i_2 = i_3 = i$, whereas $i_1 = i - 1$ (see next slide for details)
Use of Extendable Hash Structure: Example

<table>
<thead>
<tr>
<th>branch-name</th>
<th>h(branch-name)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brighton</td>
<td>0010 1101 1111 1011 0010 1100 0011 0000</td>
</tr>
<tr>
<td>Downtown</td>
<td>1010 0011 1010 0000 1100 0110 1001 1111</td>
</tr>
<tr>
<td>Mianus</td>
<td>1100 0111 1110 1101 1011 1111 0011 1010</td>
</tr>
<tr>
<td>Perryridge</td>
<td>1111 0001 0010 0100 1001 0011 0110 1101</td>
</tr>
<tr>
<td>Redwood</td>
<td>0011 0101 1010 0110 1100 1001 1110 1011</td>
</tr>
<tr>
<td>Round Hill</td>
<td>1101 1000 0011 1111 1001 1100 0000 0001</td>
</tr>
</tbody>
</table>

Initial Hash structure, bucket size = 2
**Example (Cont.)**

- Hash structure after insertion of one Brighton and two Downtown records

<table>
<thead>
<tr>
<th>Hash prefix</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bucket address table</td>
<td></td>
</tr>
<tr>
<td>A-217</td>
<td>Brighton</td>
</tr>
<tr>
<td>A-101</td>
<td>Downtown</td>
</tr>
<tr>
<td>A-110</td>
<td>Downtown</td>
</tr>
</tbody>
</table>

**Brighton** 0010  
**Downtown** 1010
Example (Cont.)

Hash structure after insertion of Mianus record

<table>
<thead>
<tr>
<th>hash prefix</th>
<th>bucket address table</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- 1
  - A-217                | Brighton   | 750 |
  - A-101                | Downtown   | 500 |
  - A-110                | Downtown   | 600 |
- 2
  - A-215                | Mianus     | 700 |

- Brighton 0010
- Downtown 1010
- Mianus 1100
Example (Cont.)

Hash structure after insertion of three Perryridge records

<table>
<thead>
<tr>
<th>Hash Prefix</th>
<th>Location</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Brighton</td>
<td>750</td>
</tr>
<tr>
<td>2</td>
<td>Downtown</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>Downtown</td>
<td>600</td>
</tr>
<tr>
<td>3</td>
<td>Mianus</td>
<td>700</td>
</tr>
<tr>
<td>3</td>
<td>Perryridge</td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>Perryridge</td>
<td>900</td>
</tr>
<tr>
<td></td>
<td>Perryridge</td>
<td>700</td>
</tr>
</tbody>
</table>

Values:
- Brighton: 0010
- Downtown: 1010
- Mianus: 1100
- Perryridge: 1111
Example (Cont.)

- Hash structure after insertion of Redwood and Round Hill records
Use of Extendable Hash Structure

- Each bucket $j$ stores a value $i_j$; all the entries that point to the same bucket have the same values on the first $i_j$ bits.

- To locate the bucket containing search-key $K_j$:
  1. Compute $h(K_j) = X$
  2. Use the first $i$ high order bits of $X$ as a displacement into bucket address table, and follow the pointer to appropriate bucket

- To insert a record with search-key value $K_j$
  - follow same procedure as look-up and locate the bucket, say $j$.
  - If there is room in the bucket $j$ insert record in the bucket.
  - Else the bucket must be split and insertion re-attempted (next slide.)
    - Overflow buckets used instead in some cases (as the case for Perryridge in previous example)
Updates in Extendable Hash Structure

To split a bucket $j$ when inserting record with search-key value $K_j$:

- **If $i > i_j$ (more than one pointer to bucket $j$)**
  - allocate a new bucket $z$, and set $i_j$ and $i_z$ to the old $i_j$ + 1.
  - make the second half of the bucket address table entries pointing to $j$ to point to $z$
  - remove and reinsert each record in bucket $j$.
  - recompute new bucket for $K_j$ and insert record in the bucket (further splitting is required if the bucket is still full)

- **If $i = i_j$ (only one pointer to bucket $j$)**
  - increment $i$ and double the size of the bucket address table.
  - replace each entry in the table by two entries that point to the same bucket.
  - recompute new bucket address table entry for $K_j$

Now $i > i_j$ so use the first case above.
Updates in Extendable Hash Structure (Cont.)

- When inserting a value, if the bucket is full after several splits (that is, \( i \) reaches some limit \( b \)) create an overflow bucket instead of splitting bucket entry table further.

- To delete a key value,
  - locate it in its bucket and remove it.
  - The bucket itself can be removed if it becomes empty (with appropriate updates to the bucket address table).
  - Coalescing of buckets can be done (can coalesce only with a “buddy” bucket having same value of \( i_j \) and same \( i_j - 1 \) prefix, if it is present)
  - Decreasing bucket address table size is also possible
    - Note: decreasing bucket address table size is an expensive operation and should be done only if number of buckets becomes much smaller than the size of the table.
Extendable Hashing vs. Other Schemes

- **Benefits of extendable hashing:**
  - Hash performance does not degrade with growth of file
  - Minimal space overhead

- **Disadvantages of extendable hashing**
  - Extra level of indirection to find desired record
  - Bucket address table may itself become very big (larger than memory)
    - Need a tree structure to locate desired record in the structure!
  - Changing size of bucket address table is an expensive operation

- **Linear hashing** is an alternative mechanism which avoids these disadvantages at the possible cost of more bucket overflows
Comparison of Ordered Indexing and Hashing

- Cost of periodic re-organization
- Relative frequency of insertions and deletions
- Is it desirable to optimize average access time at the expense of worst-case access time?

**Expected type of queries:**
- Hashing is generally better at retrieving records having a specified value of the key.
- If range queries are common, ordered indices are to be preferred
Index Definition in SQL

- Create an index
  
  ```
  create index <index-name> on <relation-name> <attribute-list>)
  ```
  
  E.g.: `create index b-index on branch(branch-name)`

- Use **create unique index** to indirectly specify and enforce the condition that the search key is a candidate key.

- To drop an index
  
  ```
  drop index <index-name>
  ```
Multiple-Key Access

- Use multiple indices for certain types of queries.

- Example:

  ```
  select account-number
  from account
  where branch-name = "Perryridge" and balance = 1000
  ```

- Possible strategies for processing query using indices on single attributes:

  1. Use index on `branch-name` to find accounts with branch name of Perryridge; test `balance = 1000`.
  2. Use index on `balance` to find accounts with balances of $1000; test `branch-name = "Perryridge"`.
  3. Use `branch-name` index to find pointers to all records pertaining to the Perryridge branch. Similarly use index on `balance`. Take intersection of both sets of pointers obtained.
Indices on Multiple Attributes

Suppose we have an index on combined search-key 
\((\text{branch-name, balance})\).

- With the \textbf{where} clause
  \textbf{where} \texttt{branch-name} = “Perryridge” \textbf{and} \texttt{balance} = 1000
the index on the combined search-key will fetch only records that satisfy both conditions.

  Using separate indices is less efficient — we may fetch many records (or pointers) that satisfy only one of the conditions.

- Can also efficiently handle
  \textbf{where} \texttt{branch-name} = “Perryridge” \textbf{and} \texttt{balance} < 1000

- But cannot efficiently handle
  \textbf{where} \texttt{branch-name} < “Perryridge” \textbf{and} \texttt{balance} = 1000
May fetch many records that satisfy the first but not the second condition.
Grid Files

- Structure used to speed the processing of general multiple search-key queries involving one or more comparison operators.
- The grid file has a single grid array and one linear scale for each search-key attribute. The grid array has number of dimensions equal to number of search-key attributes.
- Multiple cells of grid array can point to same bucket
- To find the bucket for a search-key value, locate the row and column of its cell using the linear scales and follow pointer
Example Grid File for account

![Diagram of a grid file with labeled rows and columns, buckets, and linear scales for branch-name and balance.]
Queries on a Grid File

- A grid file on two attributes $A$ and $B$ can handle queries of all following forms with reasonable efficiency:
  - $(a_1 \leq A \leq a_2)$
  - $(b_1 \leq B \leq b_2)$
  - $(a_1 \leq A \leq a_2 \land b_1 \leq B \leq b_2)$.

- E.g., to answer $(a_1 \leq A \leq a_2 \land b_1 \leq B \leq b_2)$, use linear scales to find corresponding candidate grid array cells, and look up all the buckets pointed to from those cells.
Grid Files (Cont.)

- During insertion, if a bucket becomes full, a new bucket can be created if more than one cell points to it.
  - Idea similar to extendable hashing, but on multiple dimensions
  - If only one cell points to it, either an overflow bucket must be created or the grid size must be increased
- Linear scales must be chosen to uniformly distribute records across cells.
  - Otherwise there will be too many overflow buckets.
- Periodic re-organization to increase grid size will help.
  - But reorganization can be very expensive.
- Space overhead of grid array can be high.
- R-trees (Chapter 23) are an alternative
Bitmap Indices

- Bitmap indices are a special type of index designed for efficient querying on multiple keys.
- Records in a relation are assumed to be numbered sequentially from, say, 0.
  - Given a number $n$ it must be easy to retrieve record $n$.
    - Particularly easy if records are of fixed size.
- Applicable on attributes that take on a relatively small number of distinct values.
  - E.g. gender, country, state, …
  - E.g. income-level (income broken up into a small number of levels such as 0-9999, 10000-19999, 20000-50000, 50000- infinity)
- A bitmap is simply an array of bits.
Bitmap Indices (Cont.)

- In its simplest form a bitmap index on an attribute has a bitmap for each value of the attribute
  - Bitmap has as many bits as records
  - In a bitmap for value v, the bit for a record is 1 if the record has the value v for the attribute, and is 0 otherwise

<table>
<thead>
<tr>
<th>record number</th>
<th>name</th>
<th>gender</th>
<th>address</th>
<th>income-level</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>John</td>
<td>m</td>
<td>Perryridge</td>
<td>L1</td>
</tr>
<tr>
<td>1</td>
<td>Diana</td>
<td>f</td>
<td>Brooklyn</td>
<td>L2</td>
</tr>
<tr>
<td>2</td>
<td>Mary</td>
<td>f</td>
<td>Jonestown</td>
<td>L1</td>
</tr>
<tr>
<td>3</td>
<td>Peter</td>
<td>m</td>
<td>Brooklyn</td>
<td>L4</td>
</tr>
<tr>
<td>4</td>
<td>Kathy</td>
<td>f</td>
<td>Perryridge</td>
<td>L3</td>
</tr>
</tbody>
</table>

Bitmaps for gender
- m: 1 0 0 1 0
- f: 0 1 1 0 1

Bitmaps for income-level
- L1: 1 0 1 0 0
- L2: 0 1 0 0 0
- L3: 0 0 0 0 1
- L4: 0 0 0 1 0
- L5: 0 0 0 0 0
Bitmap Indices (Cont.)

- Bitmap indices are useful for queries on multiple attributes
  - not particularly useful for single attribute queries
- Queries are answered using bitmap operations
  - Intersection (and)
  - Union (or)
  - Complementation (not)
- Each operation takes two bitmaps of the same size and applies the operation on corresponding bits to get the result bitmap
  - E.g. \[100110 \text{ AND } 110011 = 10010\]
  - \[100110 \text{ OR } 110011 = 110111\]
  - \[\text{NOT } 100110 = 011001\]
  - Males with income level L1: \[10010 \text{ AND } 10100 = 10000\]
    - Can then retrieve required tuples.
    - Counting number of matching tuples is even faster
Bitmap Indices (Cont.)

- Bitmap indices generally very small compared with relation size
  - E.g. if record is 100 bytes, space for a single bitmap is 1/800 of space used by relation.
    - If number of distinct attribute values is 8, bitmap is only 1% of relation size
- Deletion needs to be handled properly
  - Existence bitmap to note if there is a valid record at a record location
  - Needed for complementation
    - \( \text{not}(A=v): \ (NOT \ \text{bitmap-}A\-v) \ \text{AND} \ \text{ExistenceBitmap} \)
- Should keep bitmaps for all values, even null value
  - To correctly handle SQL null semantics for \( \text{NOT}(A=v) \):
    - intersect above result with \( (NOT \ \text{bitmap-}A\-Null) \)
End of Chapter
## Sample account File

<table>
<thead>
<tr>
<th>Account</th>
<th>Location</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-217</td>
<td>Brighton</td>
<td>750</td>
</tr>
<tr>
<td>A-101</td>
<td>Downtown</td>
<td>500</td>
</tr>
<tr>
<td>A-110</td>
<td>Downtown</td>
<td>600</td>
</tr>
<tr>
<td>A-215</td>
<td>Mianus</td>
<td>700</td>
</tr>
<tr>
<td>A-102</td>
<td>Perryridge</td>
<td>400</td>
</tr>
<tr>
<td>A-201</td>
<td>Perryridge</td>
<td>900</td>
</tr>
<tr>
<td>A-218</td>
<td>Perryridge</td>
<td>700</td>
</tr>
<tr>
<td>A-222</td>
<td>Redwood</td>
<td>700</td>
</tr>
<tr>
<td>A-305</td>
<td>Round Hill</td>
<td>350</td>
</tr>
</tbody>
</table>