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A Mutual Information-Based Metric for Identification of Nonlinear Injector Producer Relationships in Waterfloods

A.Khodaei, Hyokyeong Lee, F.Banaei-Kashani, C.Shahabi and I. Ershaghi, University of Southern California

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Abstract

In this paper we introduce a new analytical approach for management of waterfloods in heterogeneous reservoirs. The main contribution is the development of a process and metric to evaluate the pair-wise injector-producer (IP) relationships, i.e., to quantify the impact of any injection well on the neighboring producing wells. The proposed metric is particularly designed to consider the non-linearity of the IP relationship between the injection and production rates by using the Mutual Information (MI) data mining tool. Non-linearity of the IP relationship is the main challenge in quantifying this relationship and, to the best of our knowledge, this is the first time that MI is used in the petroleum literature for IP relationship identification. In addition to MI that captures the non-linear correlation in the IP relationship, our metric considers other parameters such as the distance between the IP pair as well as their relative injection and production rates, respectively. Leveraging our proposed metric, we propose a system, for optimal waterflooding with which a field engineer can automatically:

1) Identify the under-performing producers based on their performance characteristics such as wateroil ratio, gas oil ratio, and oil production rate;
2) Rank all injectors based on their impact on the under-performing producers using our proposed IP relationship identification metric;
3) Decide on optimal injection volumes for individual injectors that have the most impact on the under-performing producers and maximize the recovery factor.

The proposed technique can significantly reduce the decision-making time for the effective management of complex waterflood.

1. Introduction

In petroleum reservoirs, enabling engineers to model – and hence predict - injector-producer relationships is a key to gain maximum oil production with minimum operation costs. From studying the historical injection and production data from a reservoir, one can see that the production performance is controlled by various factors. One of the important factor, of course is the injection volumes. Examples of other factors include (but are not limited to) distance between each injector-producer pair, heterogeneities in subsurface structures, anisotropy in permeability, and rock property changesby drop in reservoir temperature. Typically, certain parameters may affect the output to a larger extent as compared with others, whereas some may have no effect on the behavior of the system at all. This complex behavior of oilfields renders the identification of the allocation factors between injectors and producers as an extremely hard problem [1].

Some existing methods provide allocation factors between injectors and producers based on the past experience, production and injection historical data and numerical simulations. These approaches are not accurate because they ignore the non-linear relationships between various parameters. In this paper, we present a novel mutual information based metric to model the non-linear relationships between injectors and producers in oilfields. Intuitively, mutual information measures the information that two separate variables share; i.e., it measures how much knowing one of these variables reduces our uncertainty about the other. In this case, we use mutual information to determine how much knowing about an injector can reduce our uncertainty about a producer. In other words, mutual information can help us quantify the dependence between injectors and producers in their non-linear relationship.

In addition to developing the aforementioned new metric, we developed an end-to-end proof-of-concept system to demonstrate the performance of our metric for analysis. This new system, dubbed Waterflood Management Tool (WMT), is developed on top of our existing data management system, ProDA [2]. WMT can help production engineers in decision making by quantifying and visualizing the relationship between each injector-producer pair. To summarize the functionalities of the WMT, this system provides a framework to identify under-performing producers, discover the injectors with the highest impact on those producers, and finally estimates injection volumes needed at each injection well. See Figure 1 for an example of a small field of producers (in red) and injectors (in blue).

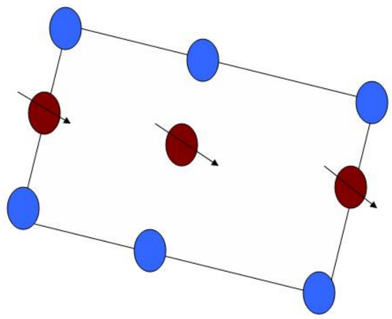

Figure 1. A field of injectors and producers

The rest of the paper is organized as follows: Section 2 briefly surveys the related work on injector-producer relationship identification problem. Section 3 first presents a short background on mutual information, and thereafter describes our proposed metric. Section 4 introduces our end-to-end system (WMT). Finally, Section 5 concludes with directions for future work.

2. Related Work

Several different techniques have been studied to address injector-producer relationship in the field of petroleum engineering. One important set of papers use Spearman rank correlations. Spearman rank correlation is an extremely convenient non-parametric statistic technique. Spearman rank correlation can be exploited to estimate the relationships between injectors and producers pair wise. Hefer et *al*. [3] calculated injector-producer relationship with the help of Spearman rank correlation method as well as knowledge of the geomechanics of the oilfields. Refunjol e*t al.* [4] also used Spearman rank correlation method to calculate injector-producer relationship. They used time series of injection and production rates an injector and nearby producers. Another work using Spearman rank correlation was done by Soeriawianata *et al.* [5]. The intuition behind their work is the fact that producers and injectors working simultaneously create both superposition and noise. Although the Spearman rank correlation is an easy-to-use and distribution-free statistic method, it does not always create correct correlations. There might be cases where it gives false negative correlations.

Another set of papers addressing the injector-producer relationship use Multiple Linear Regression methods. Alborteni *et al*. [6] used a multiple linear regression method to calculate connectivity between wells by the use of recursive model. The linear model coefficients calculated by multiple linear regression specifies the relation between a producer and adjacent injectors. In one recent work, Alborteni *et al*. [6] utilized Multiple Linear Regression (MLR) method to determine the inter-well connectivity using resistive (transmissibility) model (RM). Their method covers two cases: one is when the injection rate of the field is different than the liquid production rate of the field and another one is the case when these two are almost identical. Yousef e*t al*. [7] also used multiple linear regression method. They extended Alborteni's work by introducing capacitance (compressibility) parameter to the previous model. This new model - called capacitance model - uses two coefficients. The first coefficient is the weight representing the connectivity and is based on geology and location. The second coefficient is a time constant quantifying the degree of fluid storage between wells. Approaches based on multiple linear regression assume that model's parameters are fixed and not changing over time. This is not a correct assumption because injector-producer relationship changes very often due to different underlying factors.

In another recent work, Demiryurek *et al.* [1] used sensitivity analysis to determine the injector-producer relationships by varying the injection rates, i.e., the inputs to a trained neural network model of the oilfield, and analyzing the outputs, i.e., the production rates. With their approach, they first generated a neural network to define the mapping function between each producer and its surrounding injectors based on the historical injection and production data. Next, they utilized the generated neural network model to apply sensitivity analysis in order to quantify the *significance* of the injectors on the corresponding producers. This work is most relevant to our work. Similarly, we observe the non-linear relationship between each injector-producer pair. However, we use mutual information to address the non-linearity while they used sensitivity analysis and neural network model.

3. Metric

In this section, we present and discuss our proposed metric. Because we are using mutual information in the proposed metric we first give a little background regarding mutual information to the readers.

Background

In probability theory and information theory, the mutual information (sometimes known by the archaic term transinformation) of two random variables is a quantity that measures the mutual dependence of the two variables. The most common unit of measurement of mutual information is the bit, when logarithms to the base 2 are used. Formally, the mutual information of two discrete random variables *X* and *Y* can be defined as:

$$I(X;Y) = \sum_{y \in Y} \sum_{x \in X} p(x,y) \log\left(\frac{p(x,y)}{p_1(x)\, p_2(y)}\right)$$

where $p(x,y)$ is the joint probability distribution function of *X* and *Y*, and $p_1(x)$ and $p_2(y)$ are the marginal probability distribution functions of *X* and *Y,* respectively. In the continuous case, summation is replaced by a definite double integral:

$$I(X;Y) = \int_Y \int_X p(x,y) \log\left(\frac{p(x,y)}{p_1(x)\, p_2(y)}\right) dxdy$$

where $p(x,y)$ is now the joint probability density function of *X* and *Y*, and $p_1(x)$ and $p_2(y)$ are the marginal probability density functions of *X* and *Y* respectively.

Intuitively, mutual information measures the information that *X* and *Y* share: it measures how much knowing one of these variables reduces our uncertainty about the other. For example, if *X* and *Y* are independent, then knowing *X* does not give any information about *Y* and vice versa, so their mutual information is zero. At the other extreme, if *X* and *Y* are identical then all information conveyed by *X* is shared with *Y*: knowing *X* determines the value of *Y* and vice versa.

Proposed Metric

There are number of different factors determining the dependence between an injectors and a producer. Among the, most important factors are injection rate at the injectors and production rate of the producers. There are other factors (such as distance between the injector and producer) that we need to put into consideration as well.

We use mutual information in the context of the reservoir engineering to quantify the dependence between injector i with water injection rate X and producer j with oil production rate Y. We denote this value as MI_{ij} and define it as follows:

$$MI_{ij} = \int_Y \int_X p(x,y) \log\left(\frac{p(x,y)}{p(x)\, p(y)}\right) dxdy$$

Mutual information is of the basis to define the final metric in order to captures the non-linear correlation in the IP relationships. In addition, we also consider other factors affecting the IP relationship such as distance between the IP pair or relative injection and production rates. For example, one of the parameters affecting the IP relationship is the distance between the IP pair. We define distance between injector I and producer P as $d_{ij} = distance(i,j)$ and use it in the final metric calculation (see Equation 1). Also, since for each IP relationship, we want to consider the contribution of injector to the producer as well, we add their relative rates ($\frac{P_j}{I_i}$) to the final metric as well (see Equation 1).

After considering all the above parameters, we define score of injector i over producer j (after normalization) as :

$$s_{ij} = \frac{\overline{MI}_{ij}}{\overline{d}_{ij}^{\alpha}} \times \frac{\overline{P}_j}{\overline{I}_i} \qquad (1)$$

Basically, s_{ij} shows how much production of producer i is affected by the injection in injector j. Higher value for s_{ij} show higher dependence between producer i and injector j.

We also want to see the effect of each injector over all the producers in any section. Therefore we define global score of injector i over all the producers in the section (j € J) as follows:

$$S_i = \sum s_{ij} \qquad (2)$$

We call s_{ij} local score (ranking) of injector I and S_i global score (ranking) of injector i.

4. Waterflood Management Tool

Leveraging our proposed metric, we have developed an end-to-end system (and framework) for optimal waterflood management. This system is called Waterflood Management Tool (WMT). WMT can be potentially used by field engineers and can help them reduce decision making time by supporting following steps:

Identify the Under-Performing Producers

This first step is data driven and based on the domain knowledge. We do not integrate our metric to the system yet. Intuition behind this step is the fact that for maximizing the production in the oil field, we need to determine the weak producers first and then try to increase their productions.

There are different parameters that show which producers are under performing (are weak in other words). For instance, if the gas oil ratio (GOR) of a producer is larger than a specified threshold (solution GOR) or if the difference between its water oil ratio (ΔWOR) over two consecutive months is more that a threshold, then we can say that this producer is under performing. The factors that we put into consideration are Barrels of Oil Per Day (BOPD), Water Oil Ratio (WOR), and Gas Oil Ratio (GOR). We investigate production rate change between two consecutive months then identify producers that satisfy following conditions:

ΔBOPD < a
ΔWOR > c
ΔGOR > b
GOR $\geq$ d

as under-performing (weak) producers. Note that a,b,c,d are the thresholds defined by the user.

Rank the Injectors

In this step we rank all the injectors based on their impact on the under-performing producers using our proposed metric. After finishing with step 1, we now can have the top-*k* weak producers. We now apply our metric (Equation 1) for all those producers. The result is the local score of all possible injector-(weak) producer pair. Then, we use Equation 2 to calculate the global score of each injector. We sort the injectors based on their global ranking and then select top-*m* of them for the next step.

Deciding Optimal Injection Volumes

By now, we have two set of wells. One group is the top-k under-performing producers calculated in step 1. The other one is top-m injectors having the most impact on the producers in the first group. The last and final step is to decide what the optimal injection volumes should be for these injectors. Each injector's actual injection volume depends directly on the global score of that injector.

5. Conclusion

The main contribution of this paper is proposing a new metric for addressing the non-linearity in injector-producer relationship. We use mutual information for the first time to quantify the impact of an injector on the producing wells. We also introduce an end-to-end system to use our proposed metric.

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