



Location Encoding

-- building wheels of GeoAI

CSCI 587 Lecture 25

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Thomas Lord
Department of Computer Science

USC Viterbi

USC Viterbi

School of Engineering
Integrated Media Systems Center

What is/Why representation learning



- Data structure.
 - Arrays
 - Strings

Abstract. We present a method that achieves state-of-the-art results for synthesizing novel views of complex scenes by optimizing an underlying continuous volumetric scene function using a sparse set of input views. Our algorithm represents a scene using a fully-connected (non-convolutional) deep network, whose input is a single continuous 5D coordinate (spatial location (x, y, z) and viewing direction (θ, ϕ)) and whose output is the volume density and view-dependent emitted radiance at that spatial location. We synthesize views by querying 5D coordinates along camera rays and use classic volume rendering techniques to project the output colors and densities into an image. Because volume rendering is naturally differentiable, the only input required to optimize our representation is a set of images with known camera poses. We describe how to effectively optimize neural radiance fields to render photorealistic novel views of scenes with complicated geometry and appearance, and demonstrate results that outperform prior work on neural rendering and view synthesis. View synthesis results are best viewed as videos, so we urge readers to view our supplementary video for convincing comparisons.



187	183	174	168	150	182	129	181	172	161	155	156
155	182	163	74	75	62	33	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
206	109	5	124	131	111	120	204	166	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	105	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	216	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	148	191	193	158	227	178	143	182	105	36	190
205	174	165	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	86	159	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	215
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
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What is/Why representation learning



- Feature space
 - n = dimension
 - m = samples

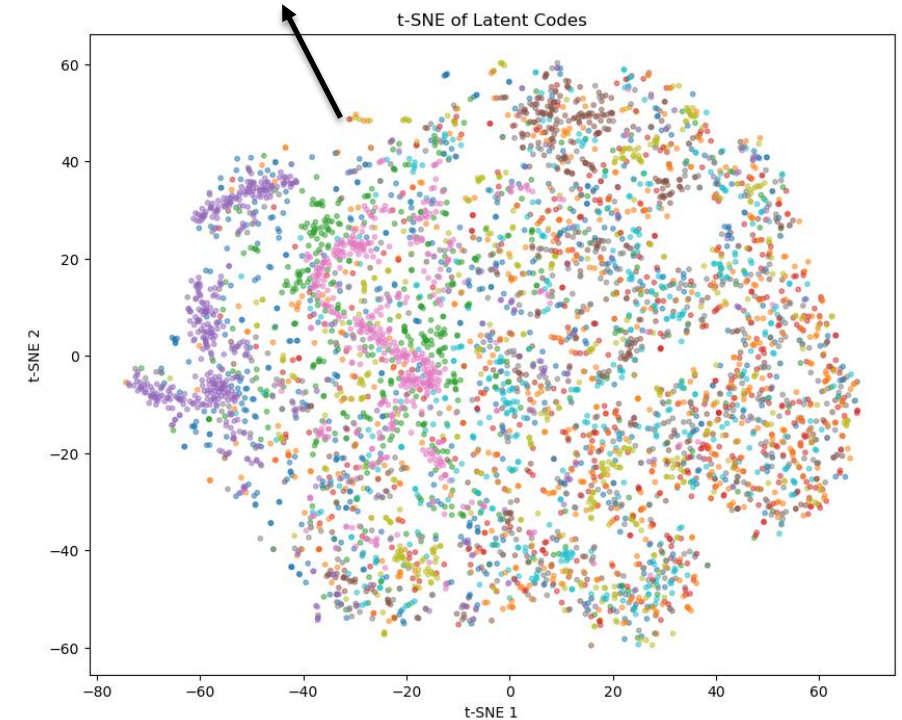
$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} 0.124 \\ \dots \dots \\ -0.940 \end{bmatrix}$$

$$v \in \mathbb{R}^{1 \times n}$$

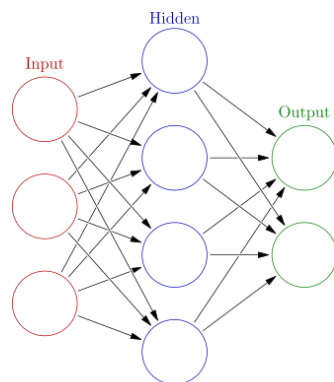
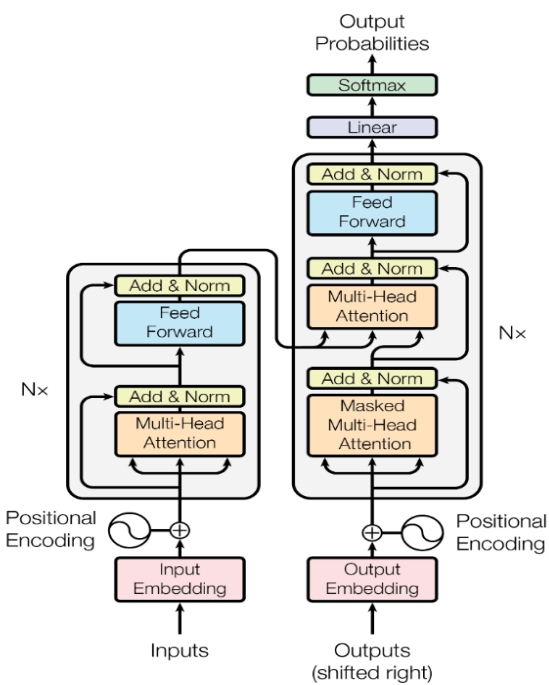
$$V \in \mathbb{R}^{m \times n}$$



What is/Why representation learning



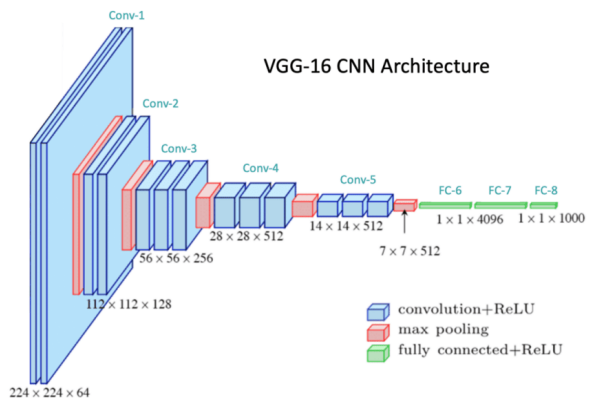
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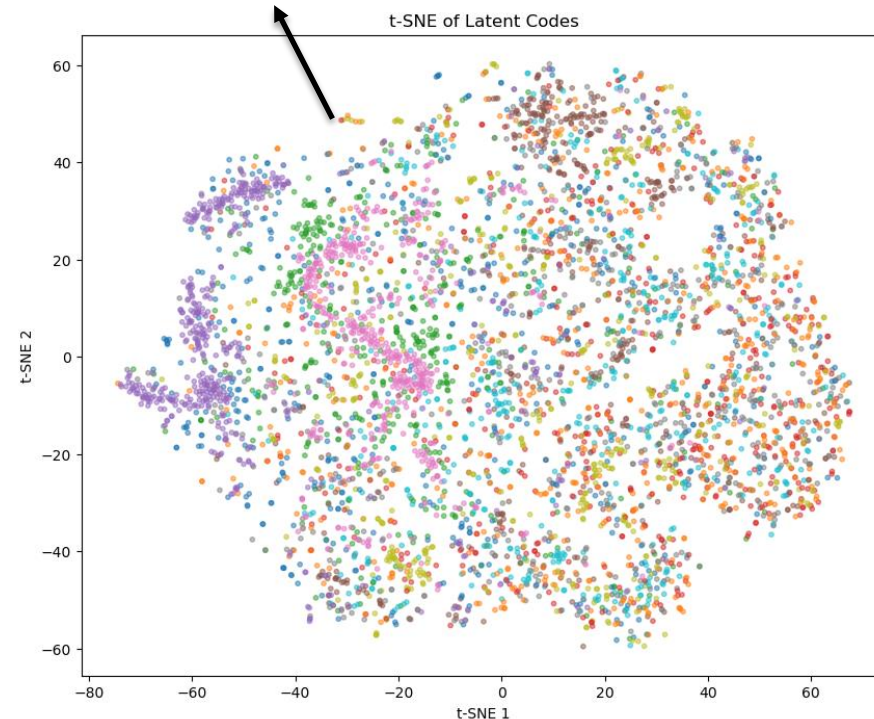
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What is/Why representation learning



- Representation learning.

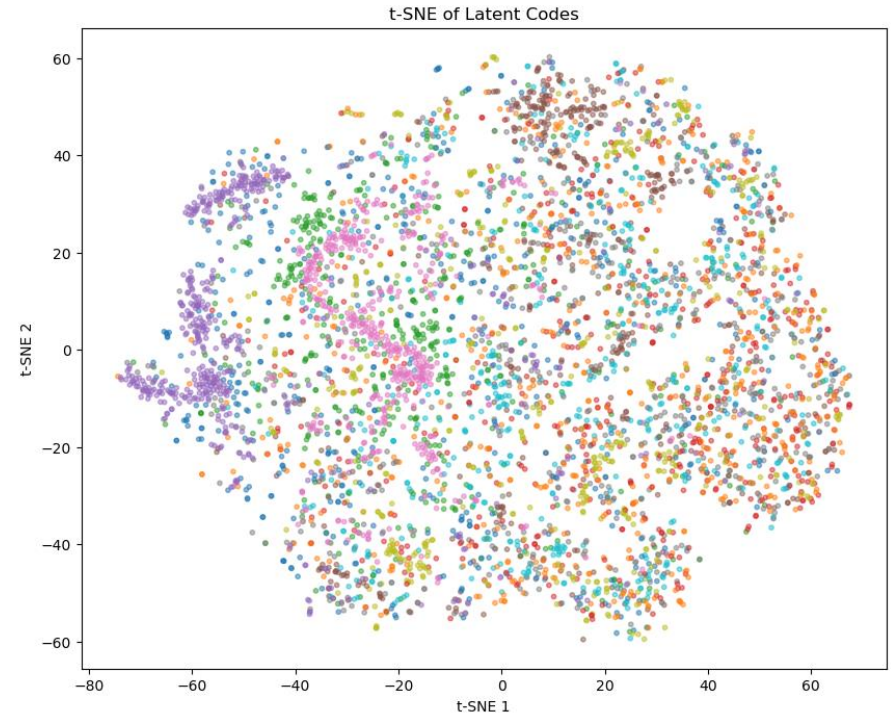
$$\theta: x \rightarrow v$$

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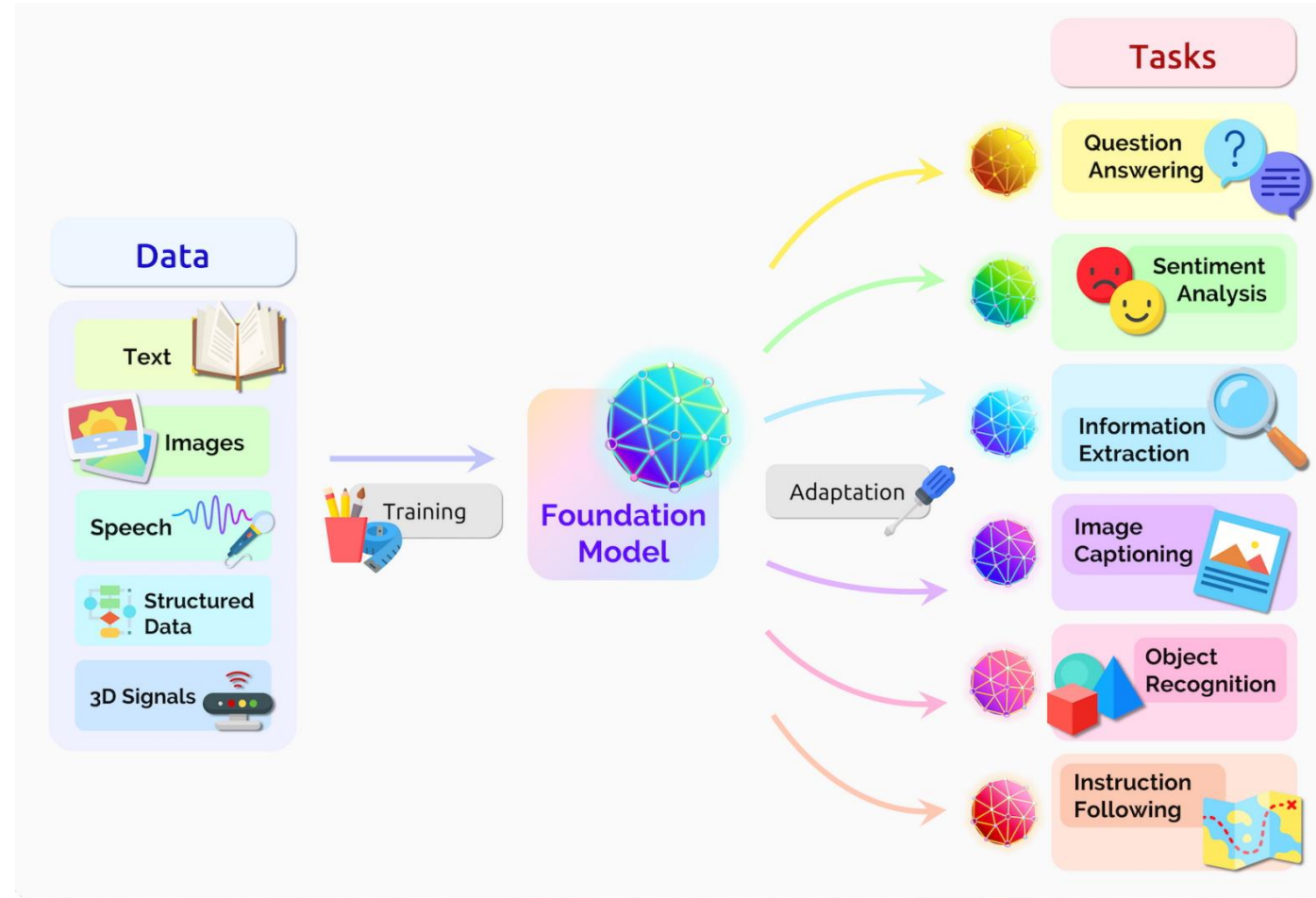
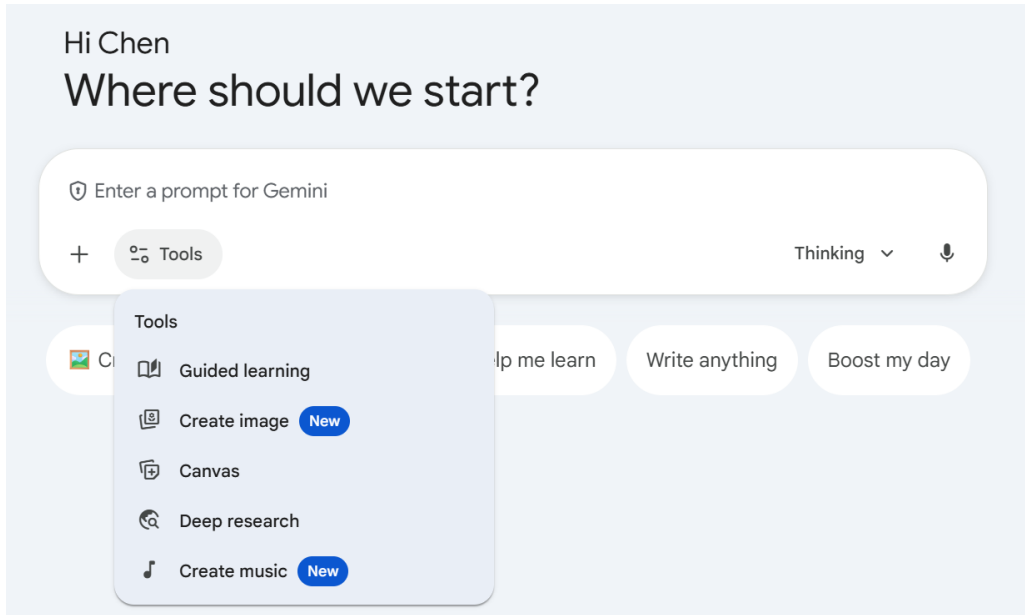


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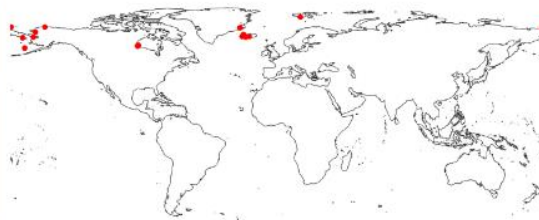
Why geospatial representation learning



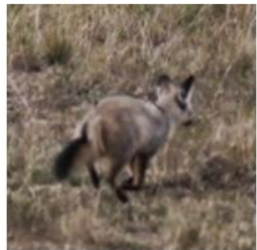
80% of all data is spatial



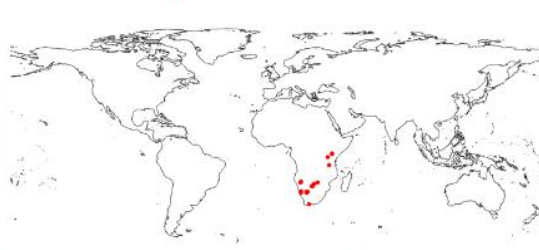
(p) Image



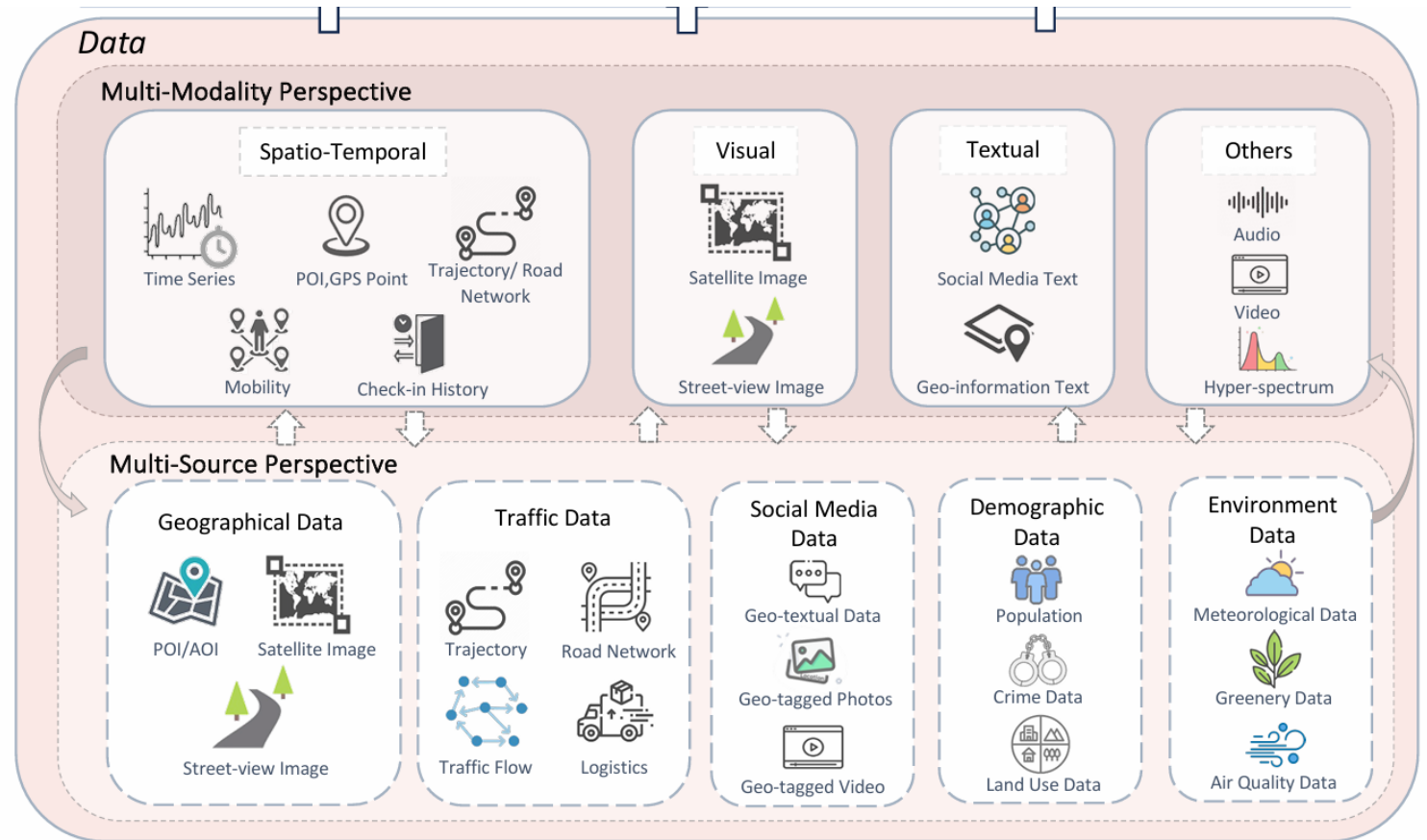
(q) Arctic Fox



(u) Image



(v) Bat-Eared Fox



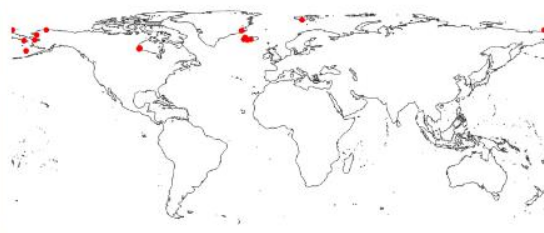
Why geospatial representation learning



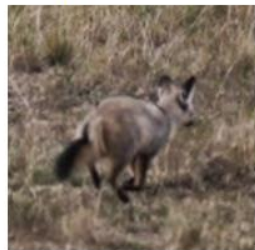
Adding location information to downstream models can boost the performance.



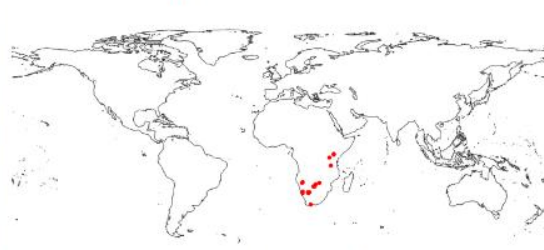
(p) Image



(q) Arctic Fox



(u) Image



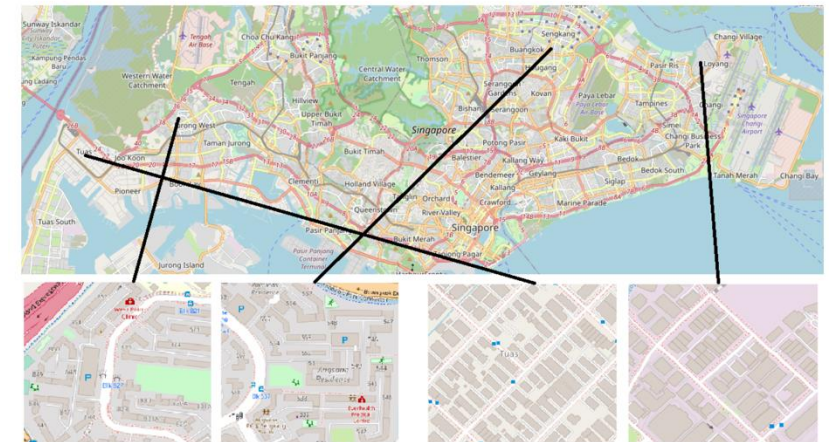
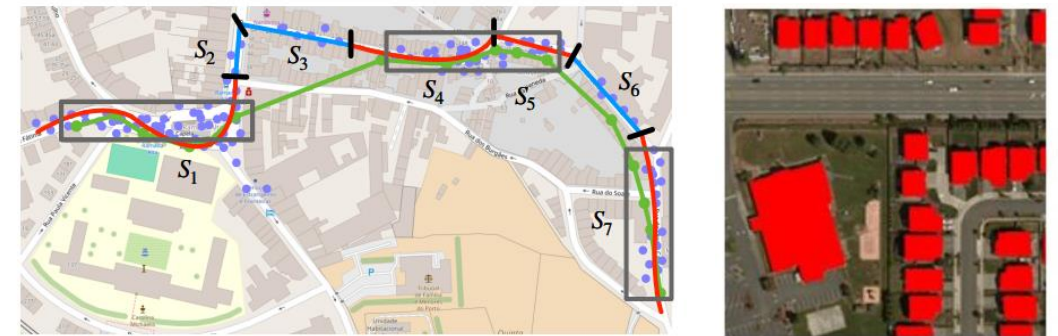
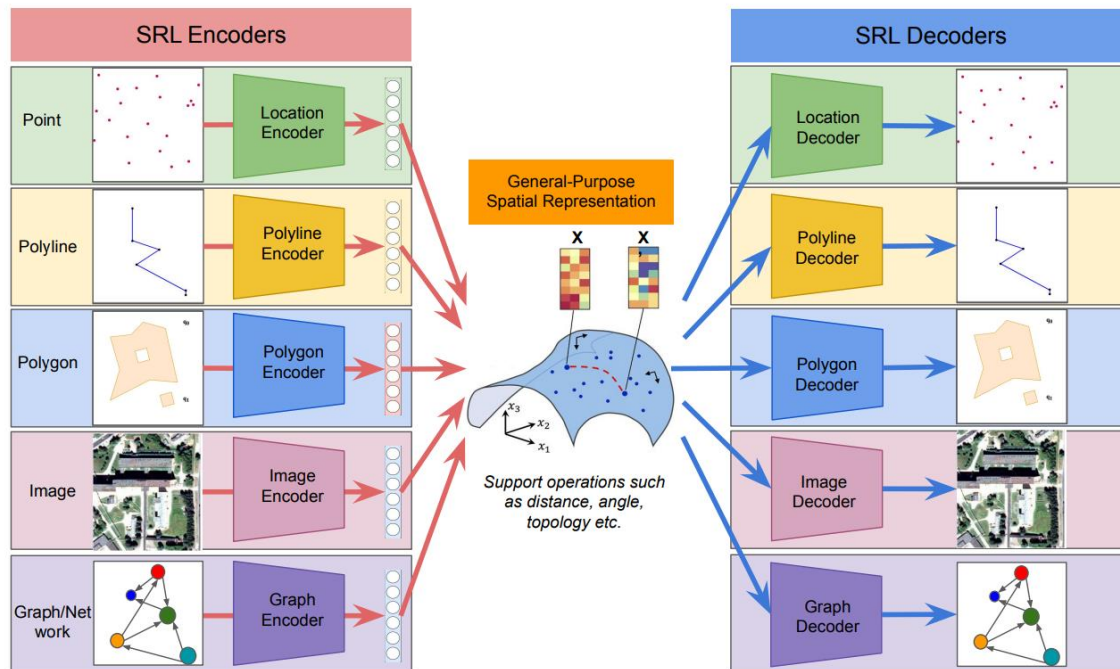
(v) Bat-Eared Fox

Task	Species Recognition						Flickr	RS
	BirdSnap	BirdSnap†	NABirds†	iNat2017	iNat2018	Avg	YFCC	fMOW
P(y x) - Prior Type	Test	Test	Test	Val	Val	-	Test	Val
No Prior (i.e. image model)	70.07	70.07	76.08	63.27	60.20	67.94	50.15	69.84
<i>tile</i> (Tang et al., 2015)	70.16	72.33	77.34	66.15	65.61	70.32	50.43	-
<i>xyz</i>	71.85	78.97	81.20	69.39	71.75	74.63	50.75	70.18
<i>wrap</i> * (Mac Aodha et al., 2019)	71.66	78.65	81.15	69.34	72.41	74.64	50.70	-
<i>wrap</i>	71.87	79.06	81.62	69.22	72.92	74.94	50.90	70.29
<i>wrap + fn</i>	71.99	79.21	81.36	69.40	71.95	74.78	50.76	70.28
<i>rbf</i> (Mai et al., 2020b)	71.78	79.40	81.32	68.52	71.35	74.47	51.09	70.65
<i>rf</i> (Rahimi et al., 2007)	71.92	79.16	81.30	69.36	71.80	74.71	50.67	70.27
<i>Space2Vec-grid</i> (Mai et al., 2020b)	71.70	79.72	81.24	69.46	73.02	75.03	51.18	70.80
<i>Space2Vec-theory</i> (Mai et al., 2020b)	71.88	79.75	81.30	69.47	73.03	75.09	51.16	70.81
<i>NeRF</i> (Mildenhall et al., 2020)	71.66	79.66	81.32	69.45	73.00	75.02	50.97	70.64
<i>Sphere2Vec-sphereC</i>	72.11	79.80	81.88	69.68	73.29	75.35	51.34	71.00
<i>Sphere2Vec-sphereC+</i>	72.41	80.11	81.97	69.75	73.31	75.51	51.28	71.03
<i>Sphere2Vec-sphereM</i>	72.06	79.84	81.94	69.72	73.25	75.36	51.35	70.99
<i>Sphere2Vec-sphereM+</i>	72.24	80.57	81.94	69.67	73.80	75.64	51.24	71.10
<i>Sphere2Vec-dfs</i>	71.75	79.18	81.39	69.65	73.24	75.04	51.15	71.46

Why geospatial representation learning



- **Spatial Representation Learning** is the first step for almost all GeoAI tasks.



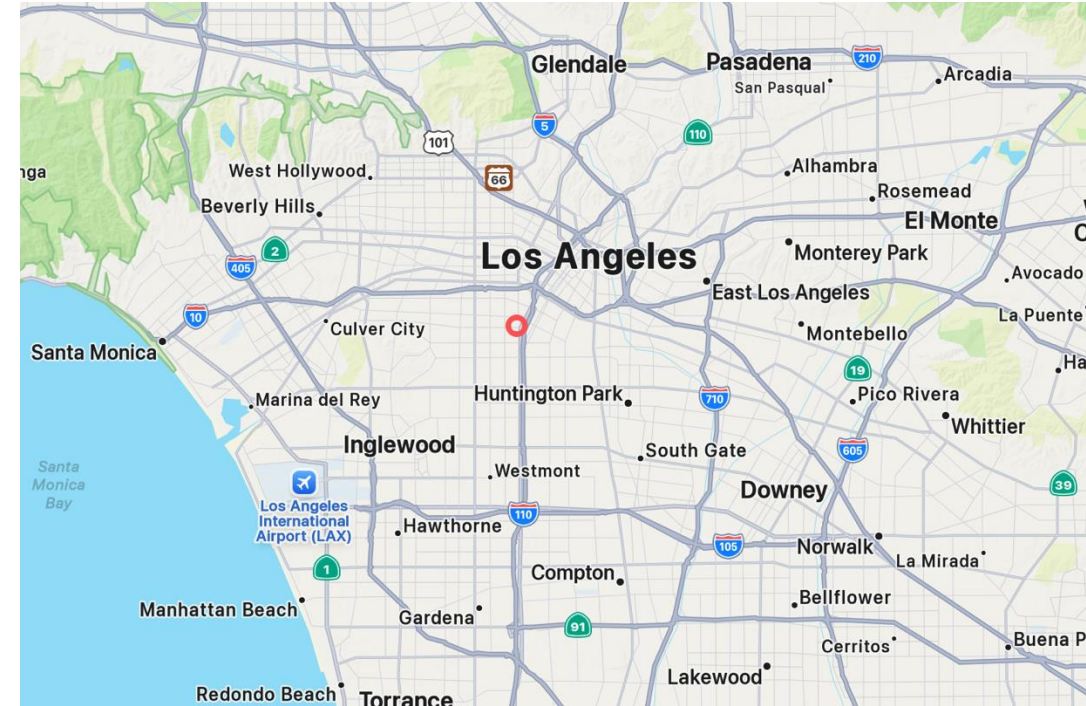
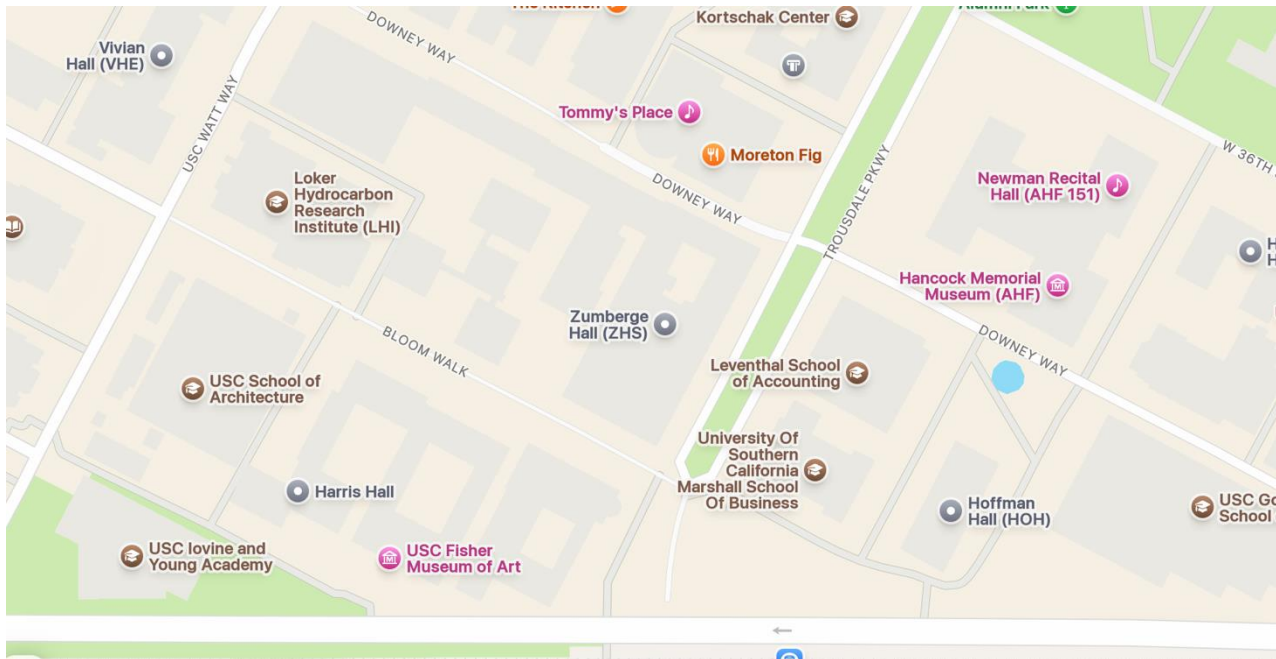
Residential Areas

Industrial Areas

Point Representation



- Scaling up

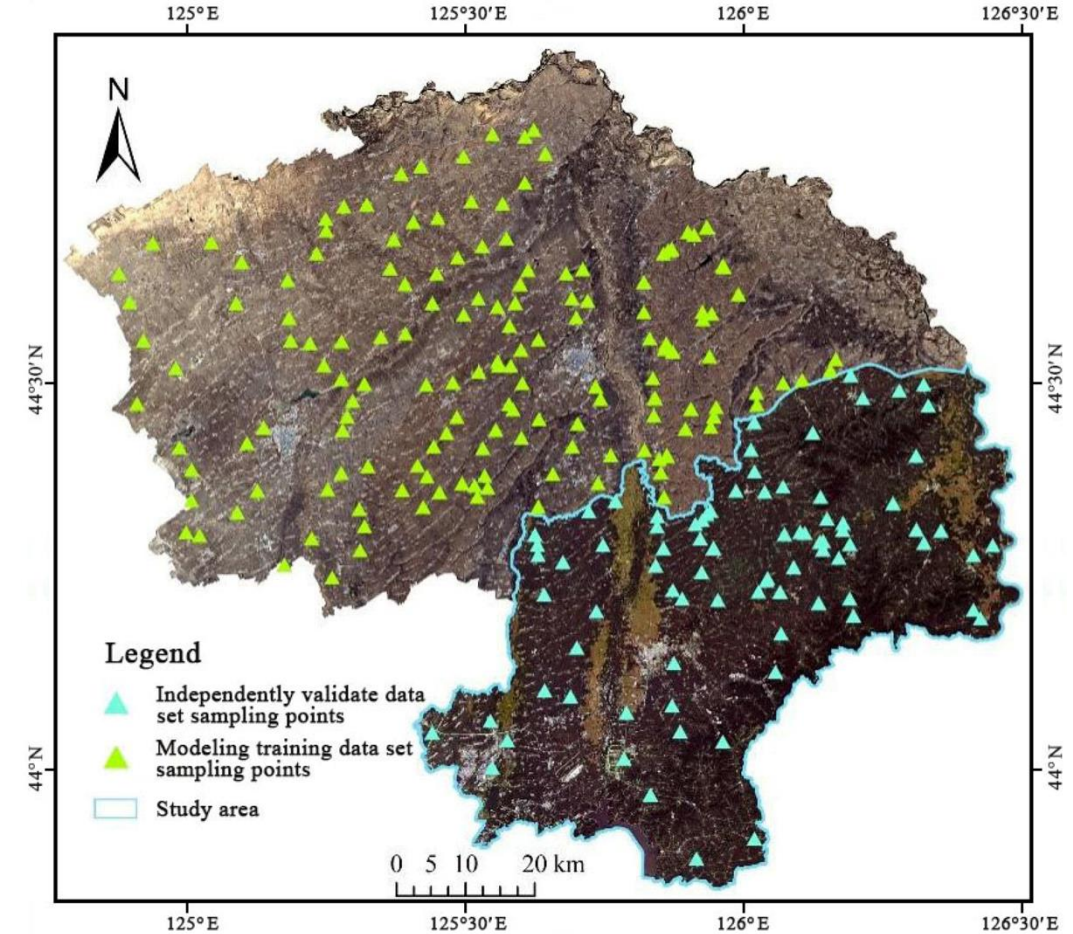




Point Representation



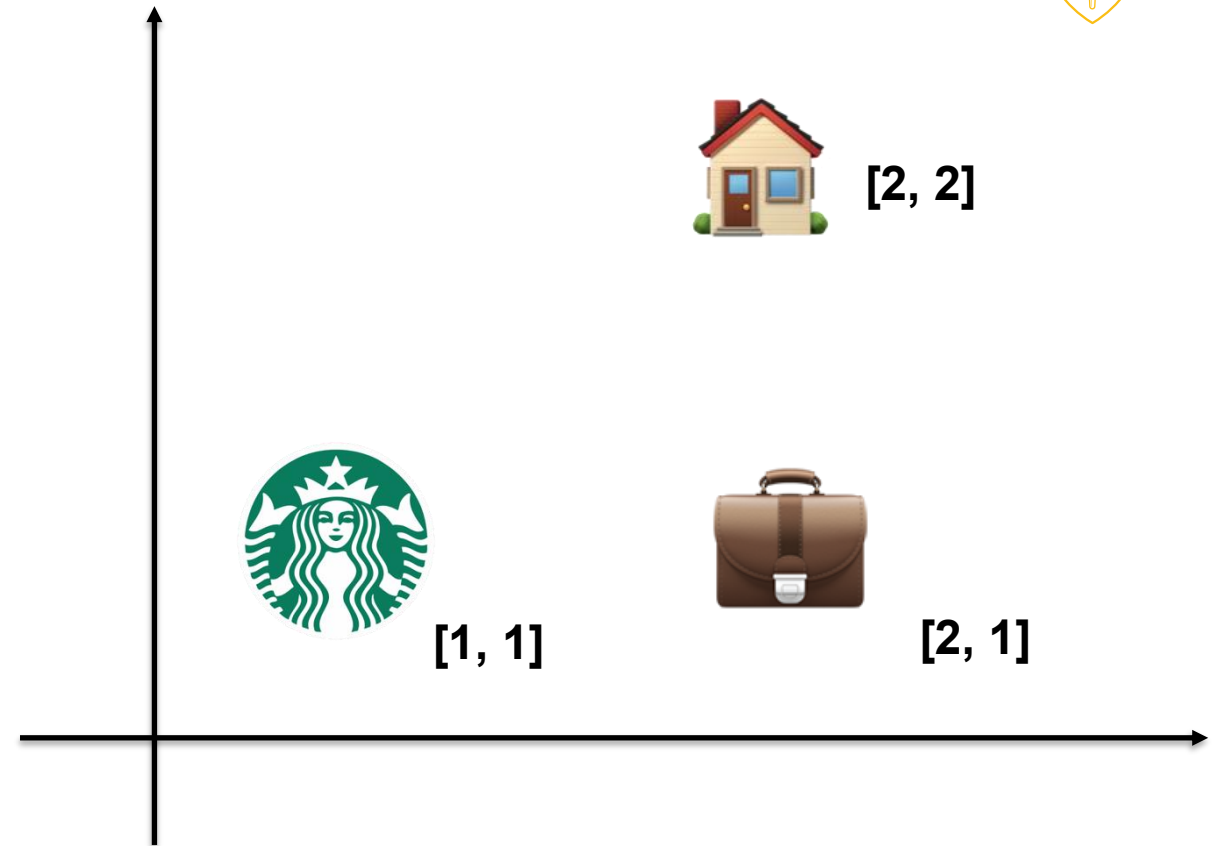
- Data is collected point by point.



Why point encoding



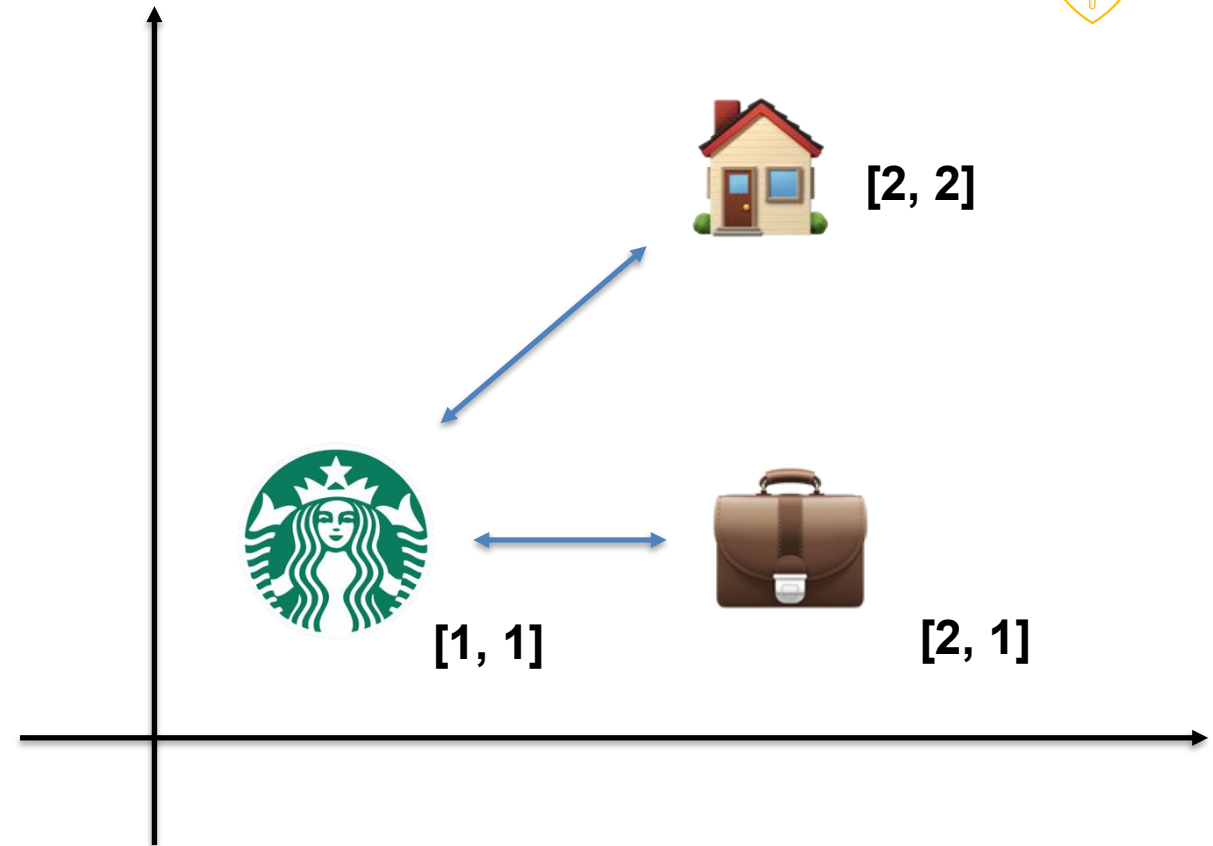
- What do we need from location:
 - Absolute position
 - Relative position
 - Positional Relationship



Why point encoding



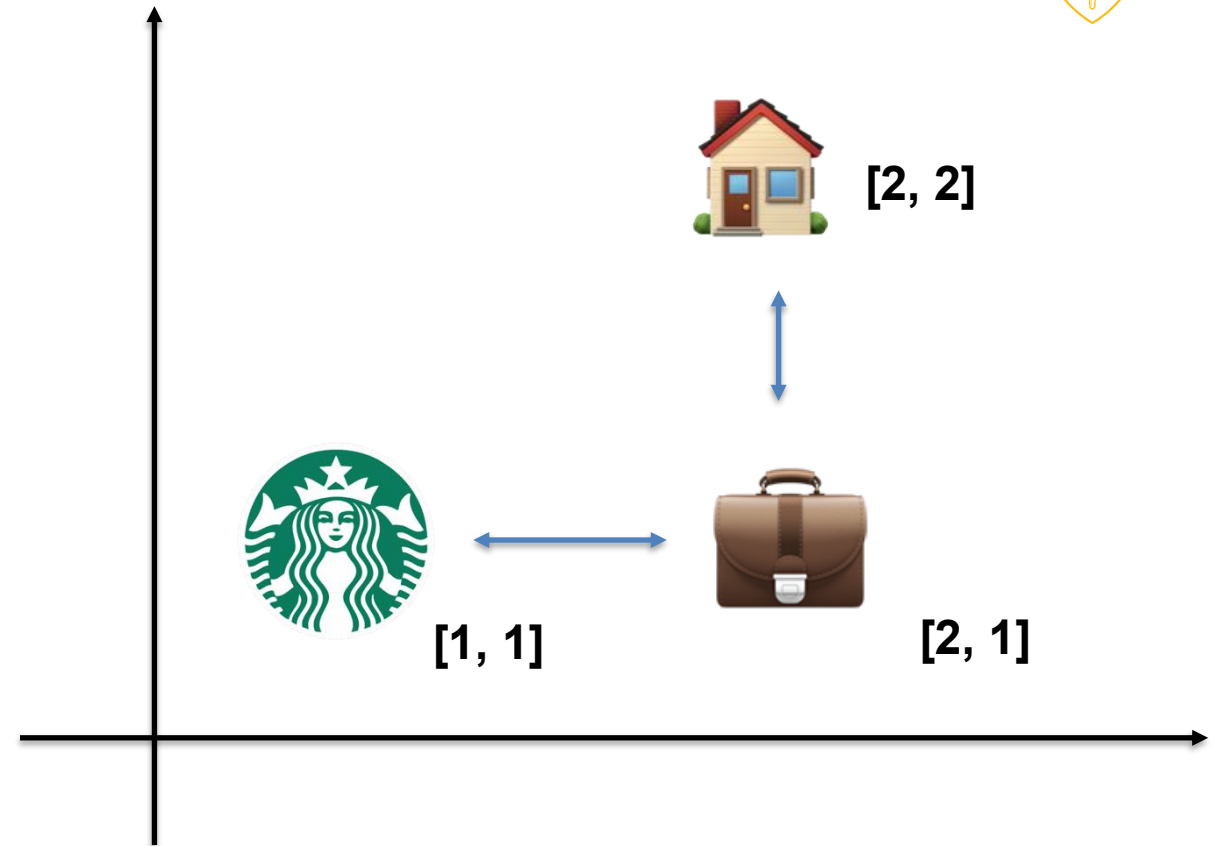
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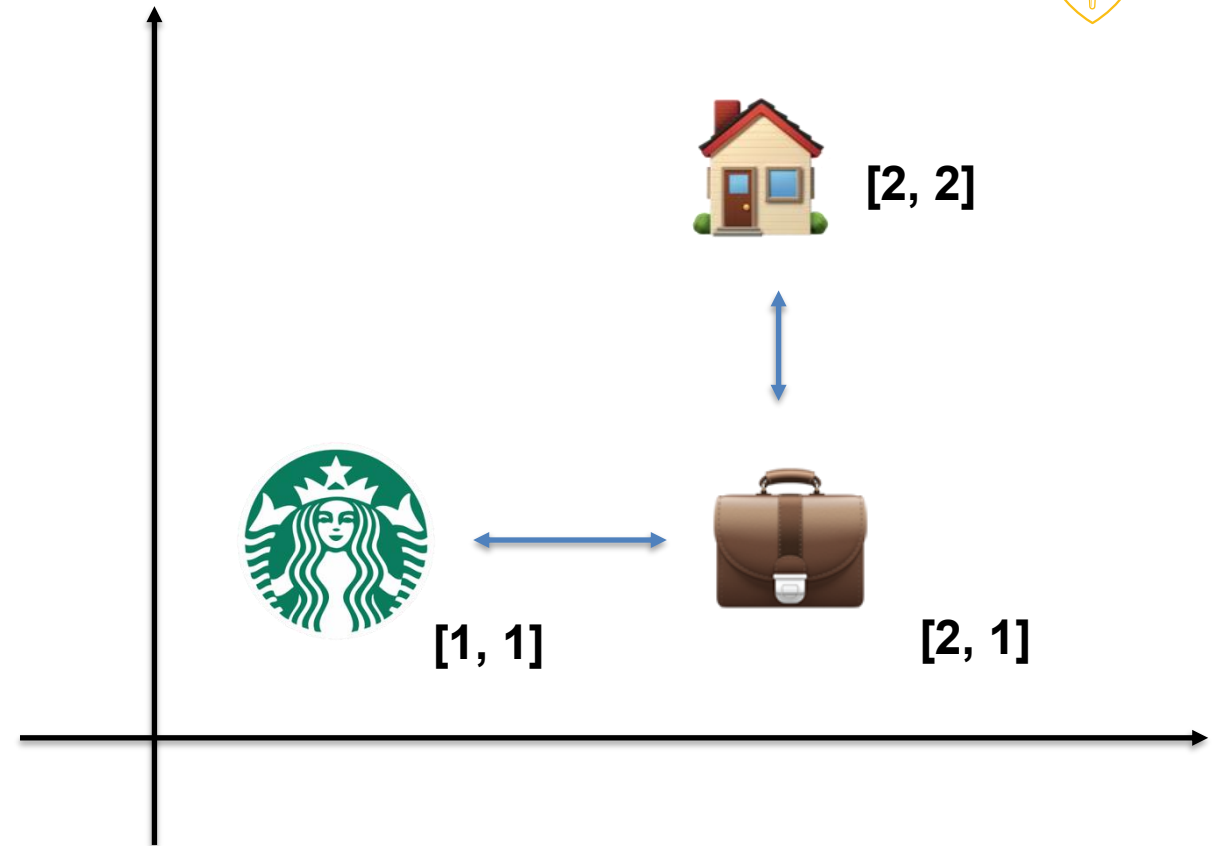
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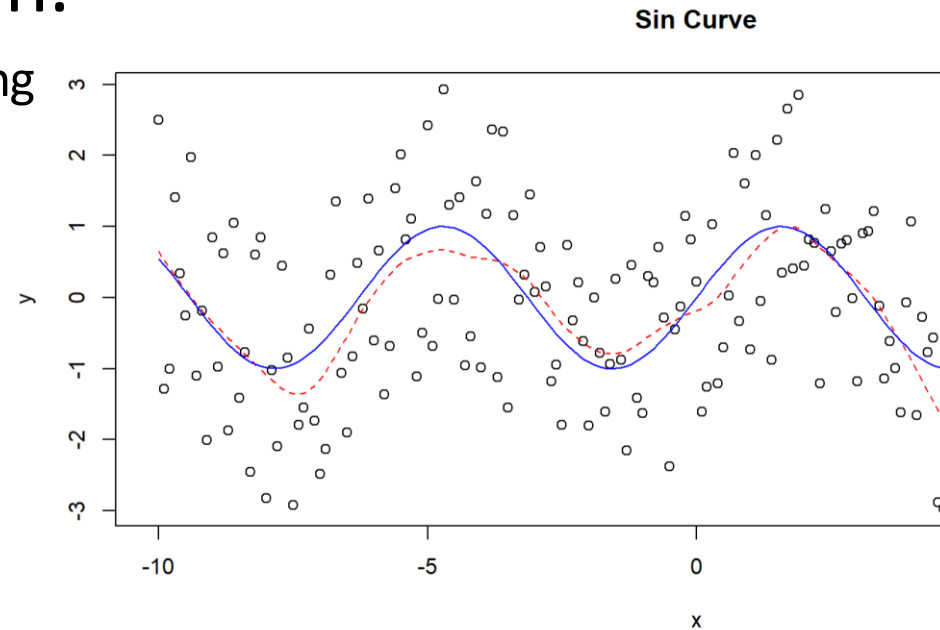
- Why not (x, y)
- What do we need from location:
 - Absolute position
 - Relative position
 - Positional Relationship



Why point encoding



- Let's start from one dimension.
 - Reduce to transformer's positional encoding
 - Absolute position
 - Relative position
 - Positional Relationship





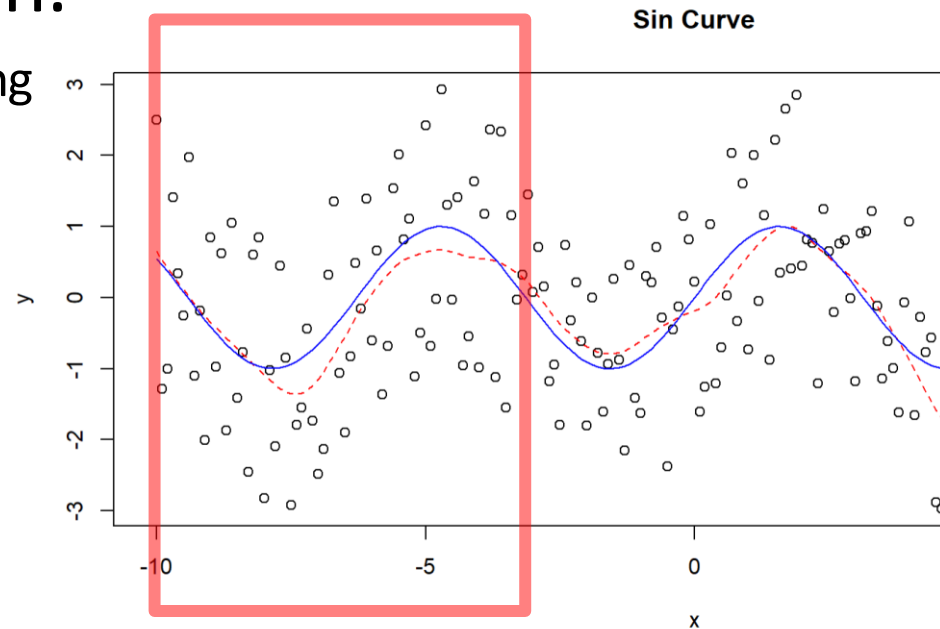
Why point encoding

- Let's start from one dimension.

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- Why not (x) (x, y)

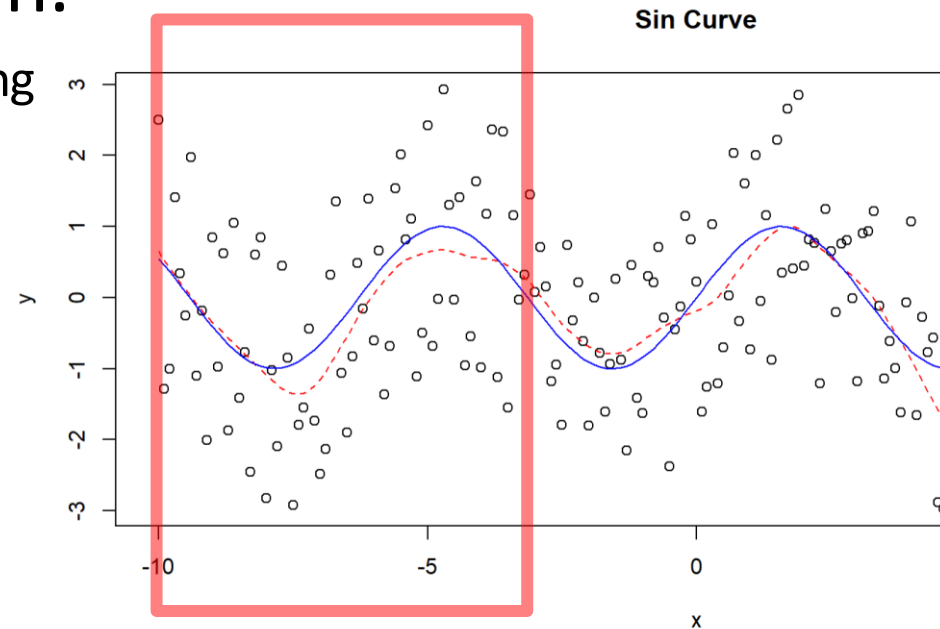
- No periodicity bias “this function should repeat” $f(-10) \rightarrow 1$ $f(-5) \rightarrow 1$ $f(0) \rightarrow ?$
- No frequency awareness “whether variation is slow or fast” $f(-10) - f(-7.5) = 2$ $f(0.12) - f(7.62) = ?$





Why point encoding

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 - Relative position
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- Why not (x, y)



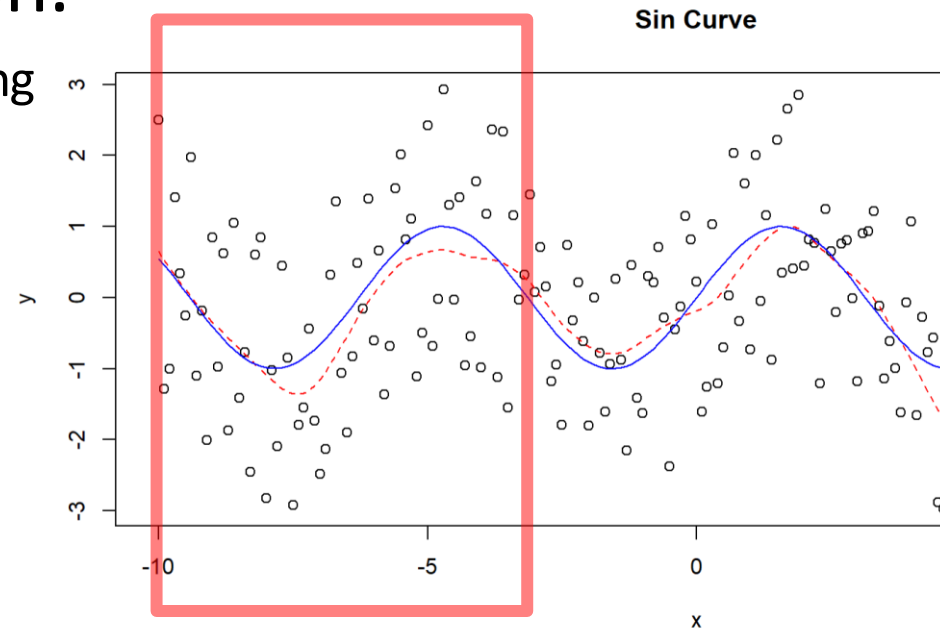
- **Lack of inductive bias** \Rightarrow High-variance model
 - Memorize *coordinates* instead of learning *generalizable patterns*.
- **Poor generalization** across scales/regions e.g. NYC vs LA

Why point encoding



Adding inductive bias

- Let's start from one dimension.
 - Reduce to transformer's positional encoding
 - Absolute position
 - Relative position
 - Positional Relationship
- What would be good?



$$f(x) \rightarrow f(\sin(x))$$

“this function should repeat”

$$y = w \sin(x)$$

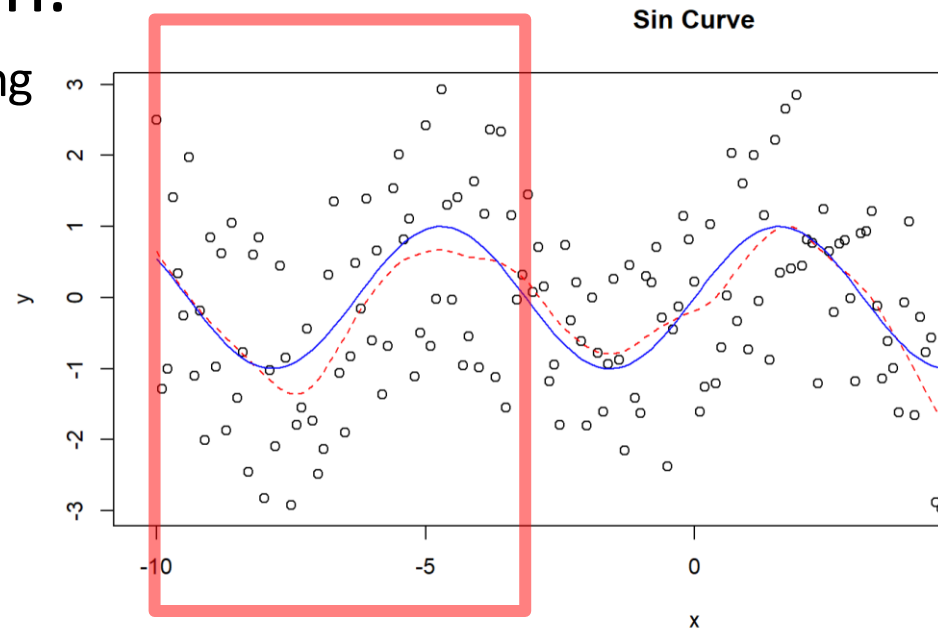
“whether variation is slow or fast”

Why point encoding



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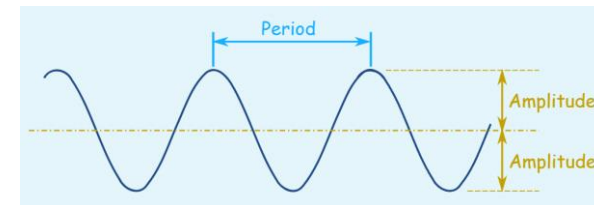
$$f(x) \rightarrow f(\sin(x))$$

“this function should repeat”

$$y = w \sin(ax)$$

“whether variation is slow or fast”

$$\text{Period} = \frac{2\pi}{a}$$

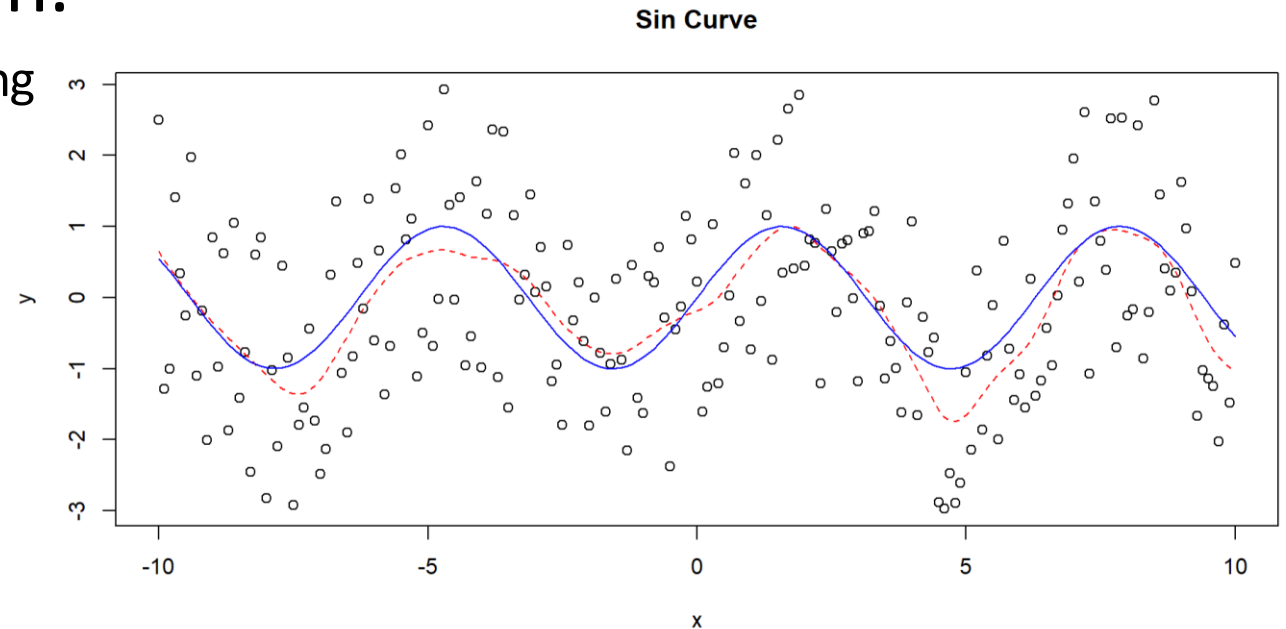


Why point encoding



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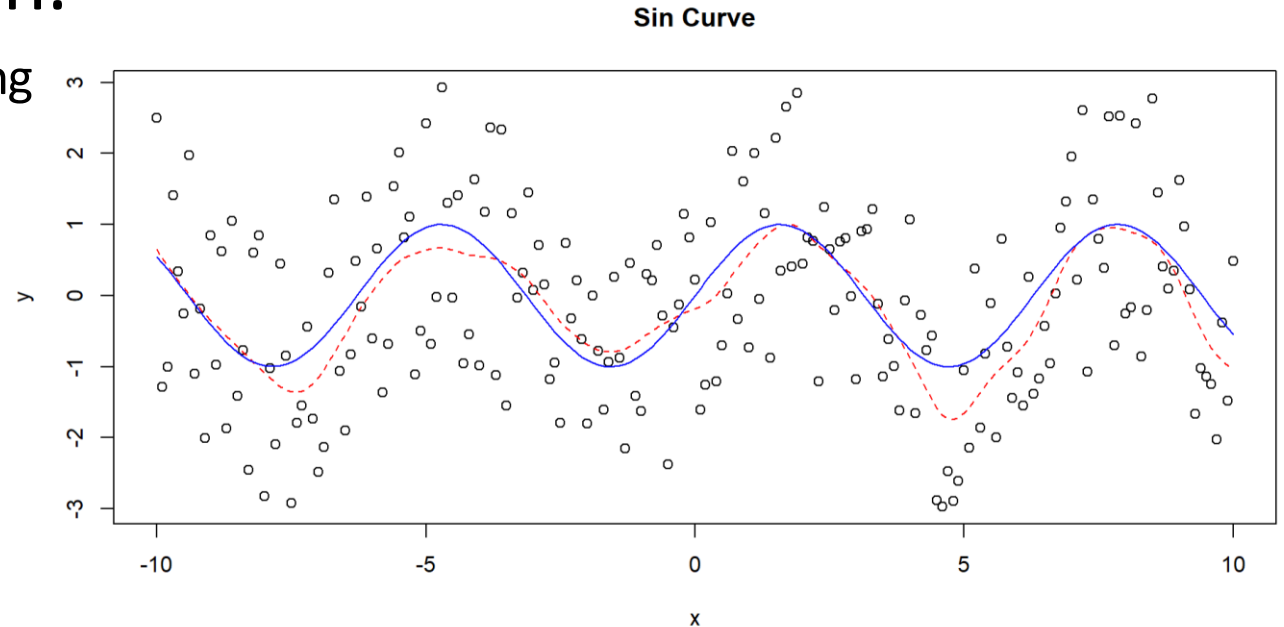


Adding a good inductive bias

- Let's start from one dimension.
 - Reduce to transformer's positional encoding
 - Absolute position
 - Relative position
 - Positional Relationship
- How do we know period **a**?

$$f(x) \rightarrow f(\sin(x))$$

$$y = w \sin(\mathbf{a}x)$$

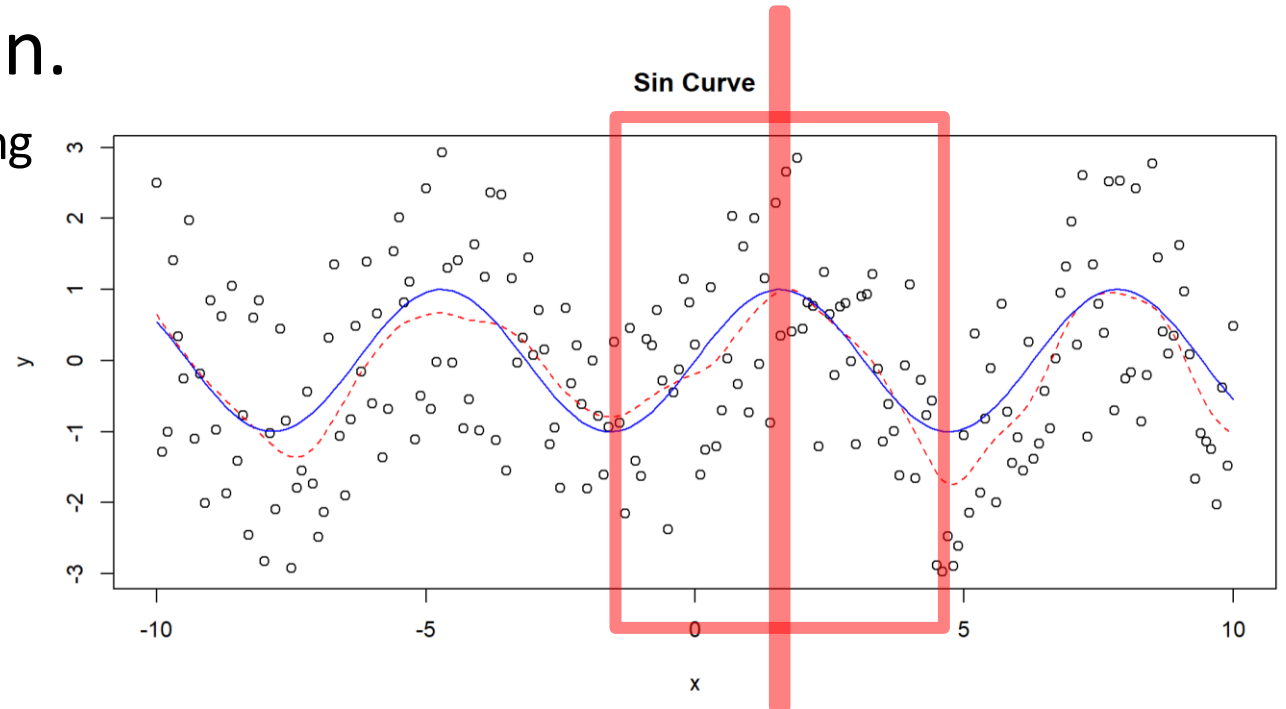


Why point encoding

Adding the prior that a geospatial phenomenon could have multiple variation patterns in different scales.



- Let's start from one dimension.
 - Reduce to transformer's positional encoding
 - Absolute position
 - Relative position
 - Positional Relationship
- How do we know period **a**?



$$x \in \mathbb{R}^{1 \times n}$$

$$f(x) \rightarrow f(\sin(x))$$

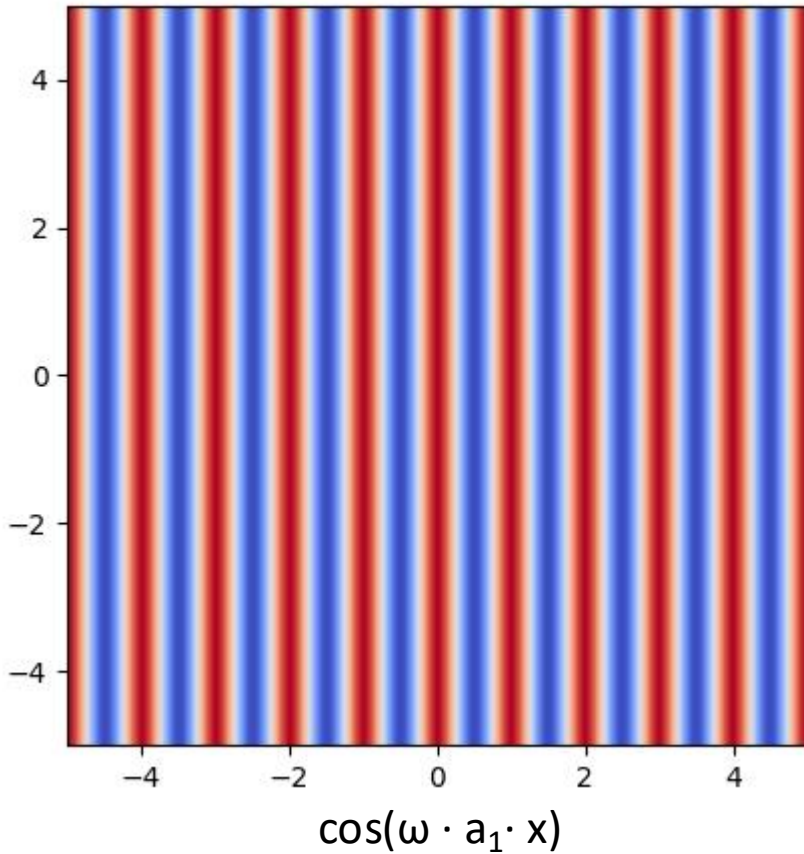
$$y = w \sin(ax)$$

$$x \rightarrow [\sin(a_1x), \sin(a_2x), \sin(a_3x), \dots, \sin(a_nx)]$$



2D space

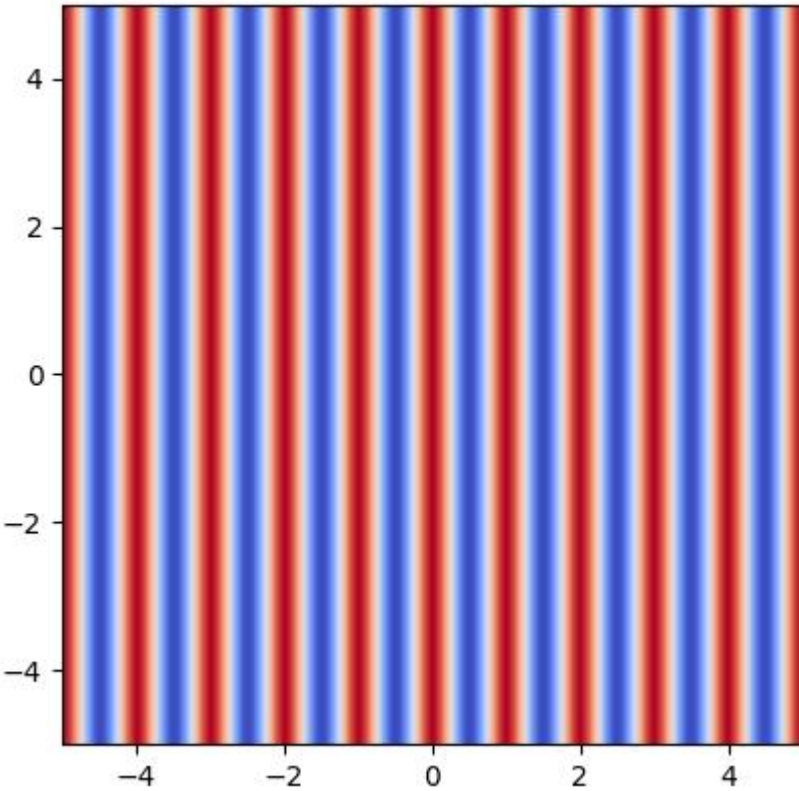
Encoding function: $f(x, y) = [\cos(\omega x), \cos(\omega y)]$ where $\vec{x} = (x, y)$, $\vec{a} =$ unit vector



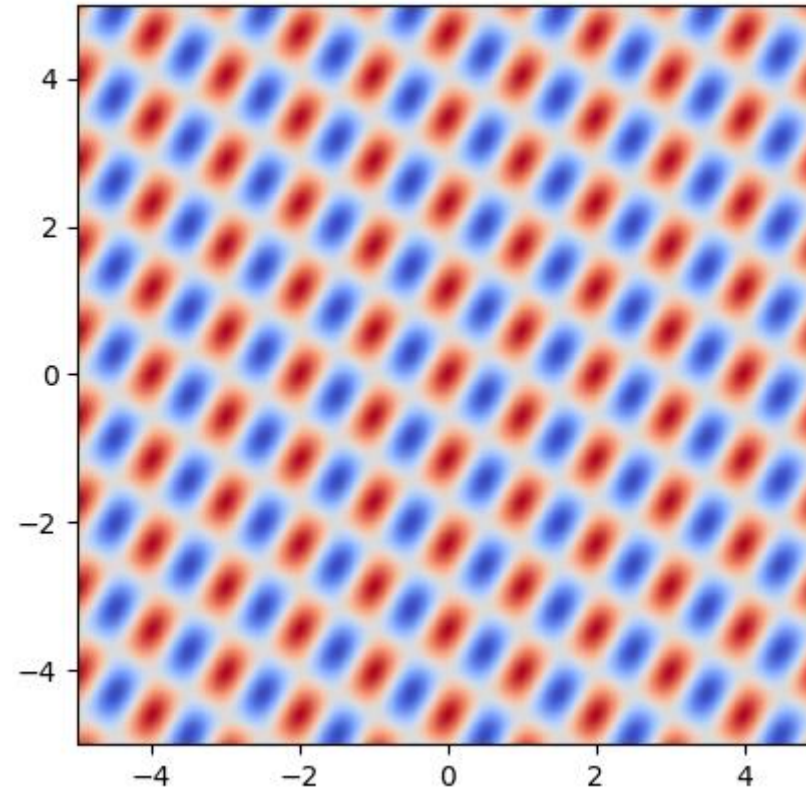


2D space

Encoding function: $f(x, y) = [\cos(\omega x), \cos(\omega y)]$ where $\vec{x} = (x, y)$, $\vec{a} =$ unit vector



$\cos(\omega \cdot a_1 \cdot x)$



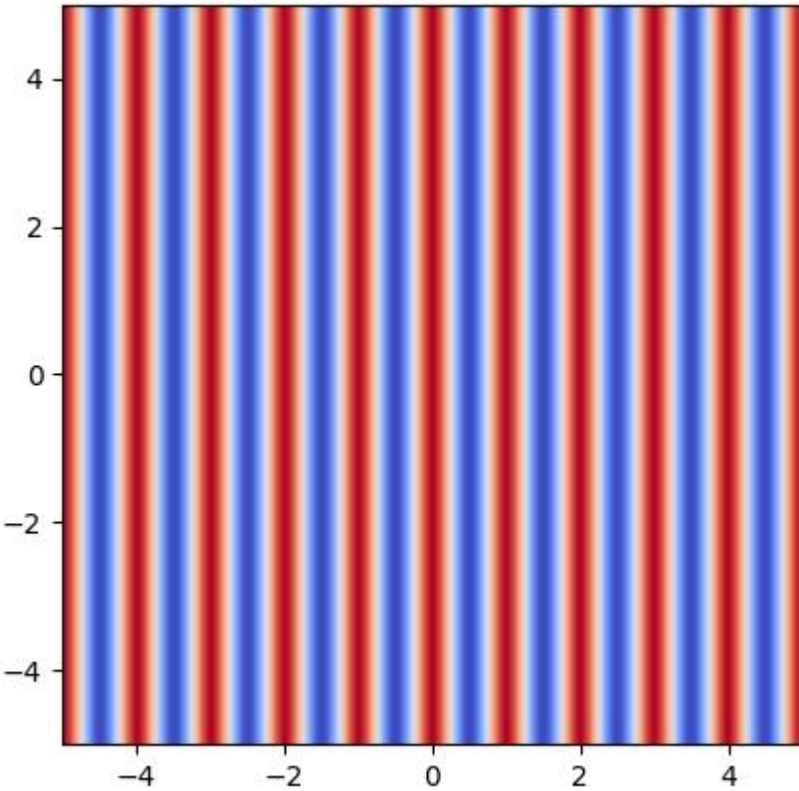
$\cos(\omega \cdot a_1 \cdot x) + \cos(\omega \cdot a_2 \cdot x)$

2D space

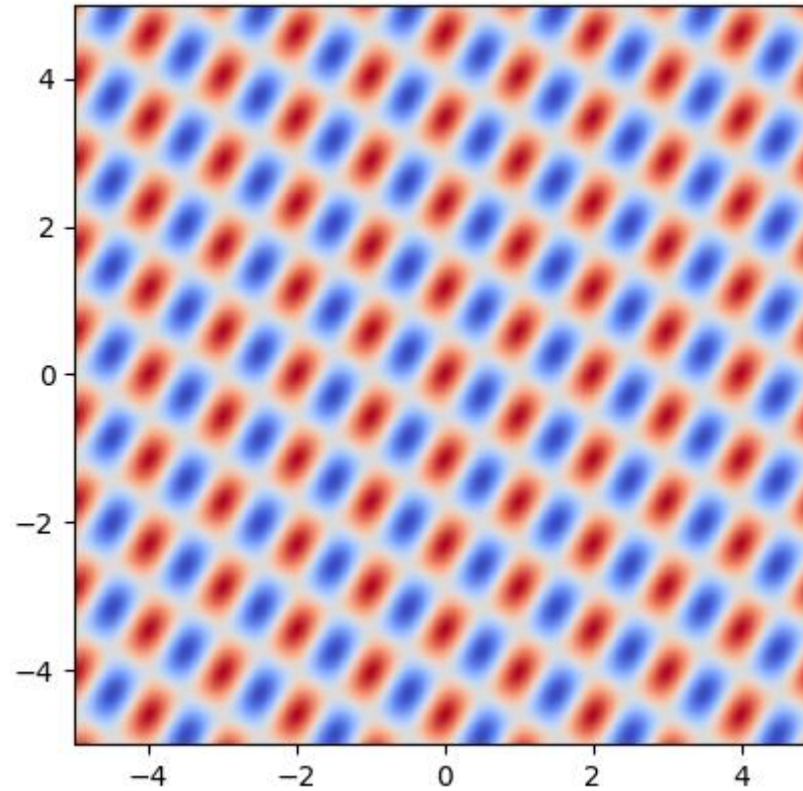
Adding the prior that a geospatial phenomenon could have multiple variation patterns in different directions.



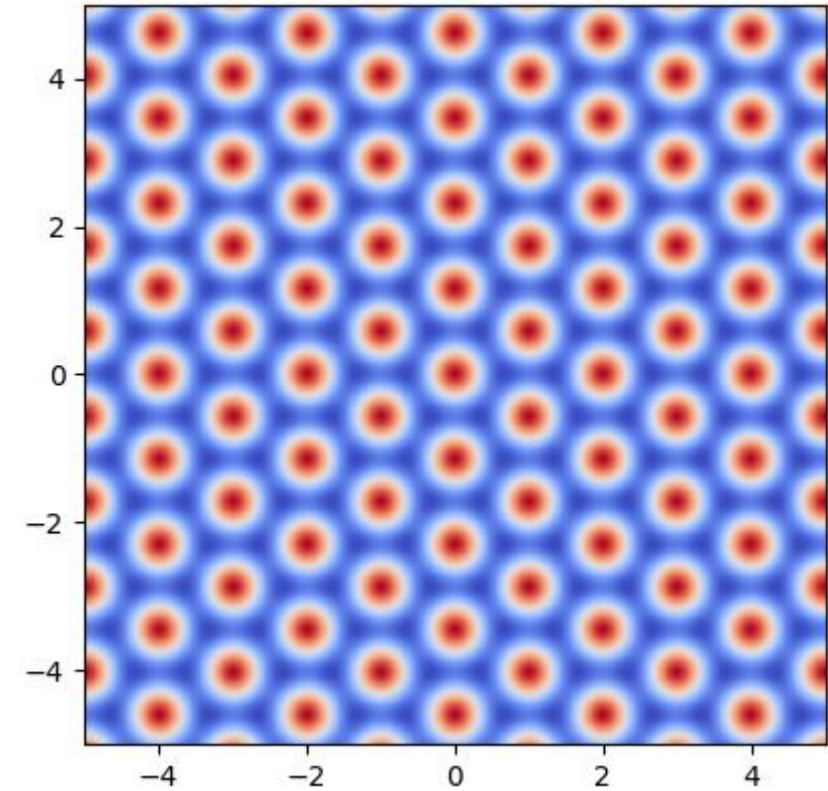
Encoding function: $f(x, y) = [\cos(\omega x), \cos(\omega y)]$ where $\vec{x} = (x, y)$, $\vec{a} =$ unit vector



$\cos(\omega \cdot a_1 \cdot x)$



$\cos(\omega \cdot a_1 \cdot x) + \cos(\omega \cdot a_2 \cdot x)$



$\cos(\omega \cdot a_1 \cdot x) + \cos(\omega \cdot a_2 \cdot x) + \cos(\omega \cdot a_3 \cdot x)$

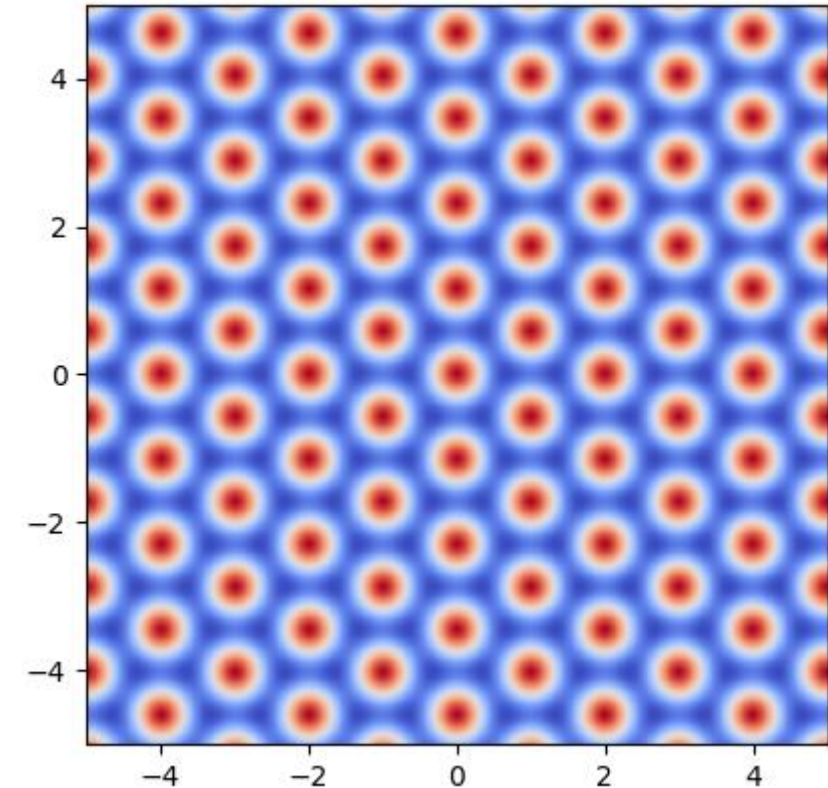
2D space



Encoding function: $f(x, y) = [\cos(\omega x), \cos(\omega y)]$ where $\vec{x} = (x, y)$, $\vec{a} =$ unit vector


$$PE^{(t)}(\mathbf{x}) = [PE_0^{(t)}(\mathbf{x}); \dots; PE_s^{(t)}(\mathbf{x}); \dots; PE_{S-1}^{(t)}(\mathbf{x})]$$

$$PE_{s,j}^{(t)}(\mathbf{x}) = \left[\cos\left(\frac{\langle \mathbf{x}, \mathbf{a}_j \rangle}{\lambda_{min} \cdot g^{s/(S-1)}}\right); \sin\left(\frac{\langle \mathbf{x}, \mathbf{a}_j \rangle}{\lambda_{min} \cdot g^{s/(S-1)}}\right) \right] \forall j = 1, 2, 3;$$



2D space

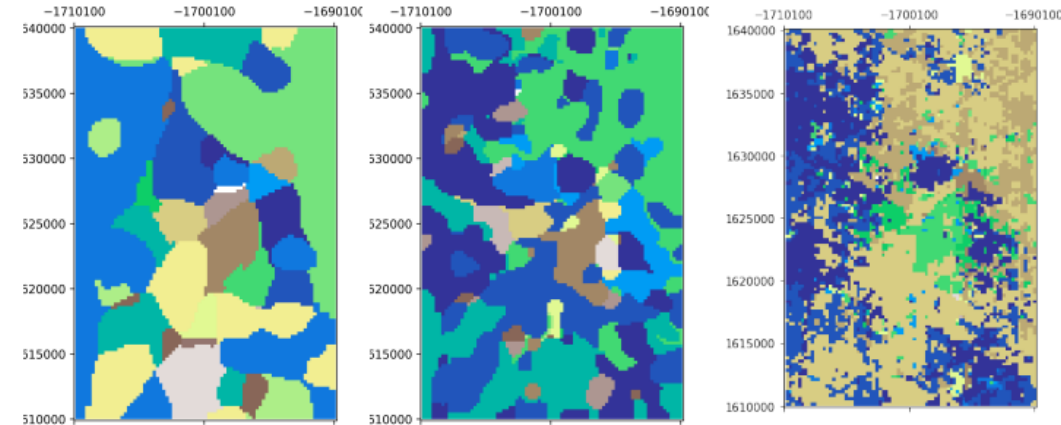
Adding the prior that a geospatial phenomenon could have multiple variation patterns in different scales.



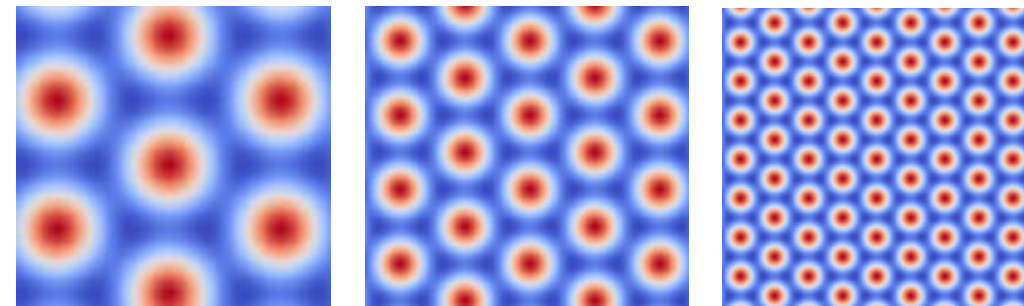
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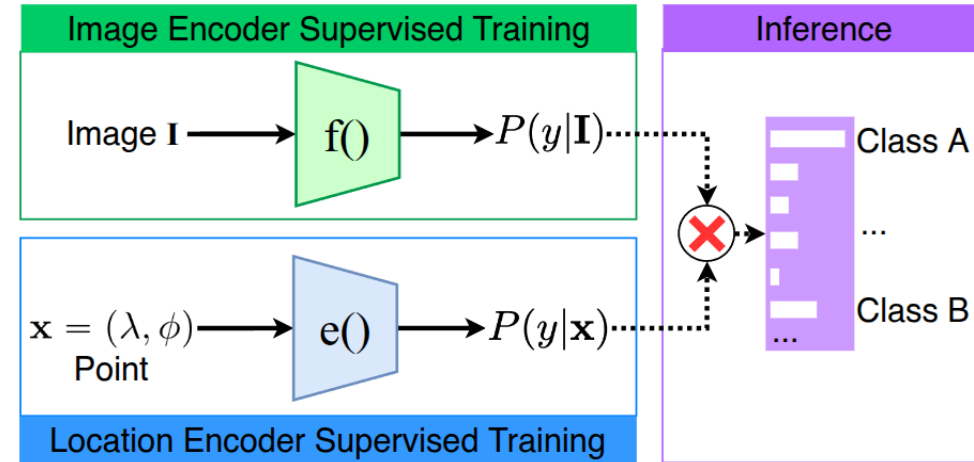
(e) $\lambda_{min}=1k$ (f) $\lambda_{min}=500$ (g) $\lambda_{min}=50$



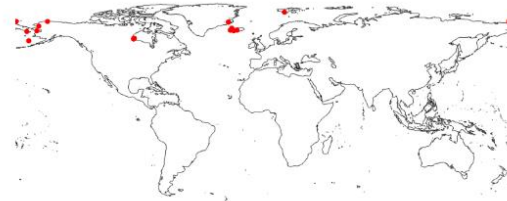
Multi-scale representation learning for spatial feature distributions using **grid cells**, ICLR 2020.

Results

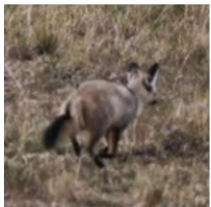
Space2Vec: Location Aware Image Classification



(a) Arctic Fox



(b) Arctic Fox Locations



(c) Bat-Eared Fox



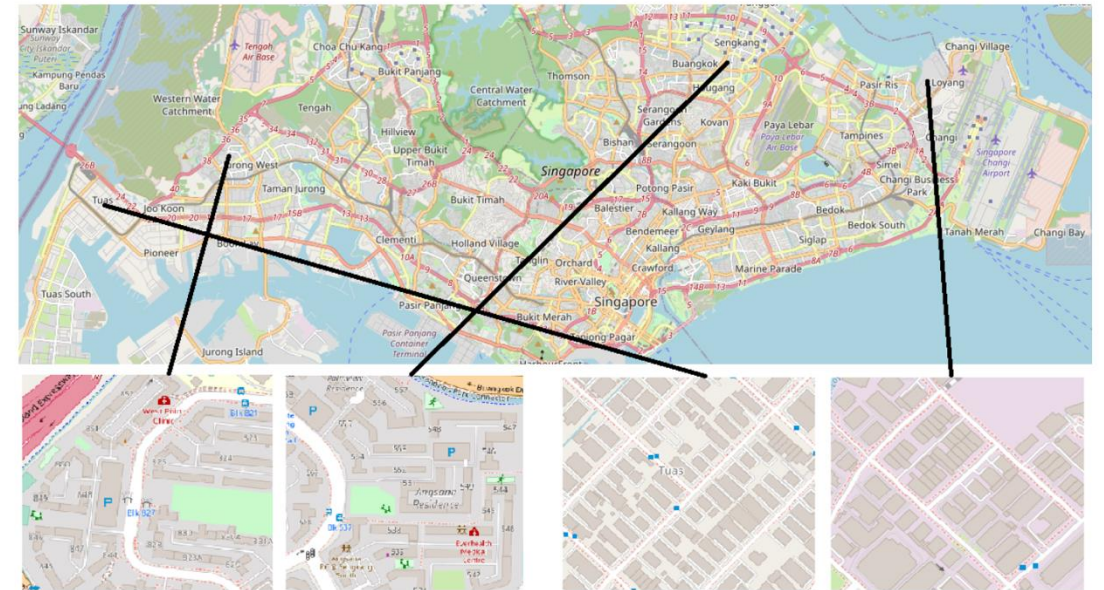
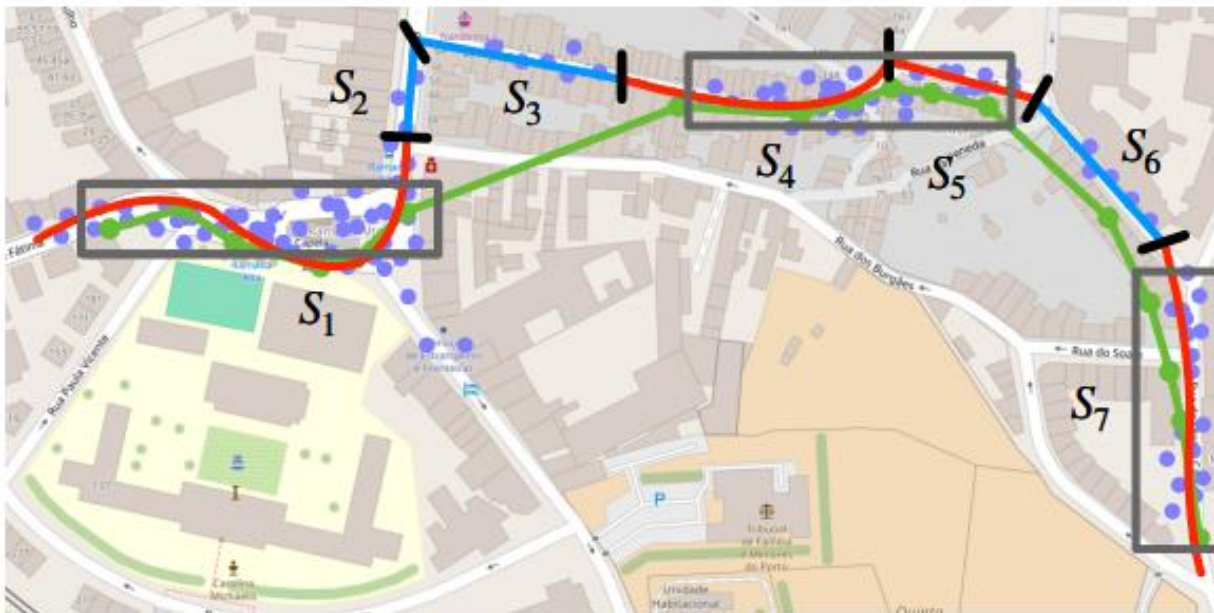
(d) Bat-Eared Fox Locations

	BirdSnap [†]	NABirds [†]
No Prior (i.e. uniform)	70.07	76.08
Nearest Neighbor (num)	77.76	79.99
Nearest Neighbor (spatial)	77.98	80.79
Adaptive Kernel (Berg et al., 2014)	78.65	81.11
tile (Tang et al., 2015) (location only)	77.19	79.58
wrap (Mac Aodha et al., 2019) (location only)	78.65	81.15
rbf ($\sigma=1k$)	78.56	81.13
grid ($\lambda_{min}=0.0001, \lambda_{max}=360, S=64$)	79.44	81.28
theory ($\lambda_{min}=0.0001, \lambda_{max}=360, S=64$)	79.35	81.59

What's next



- We need a unified representation of different types of geospatial data.



Residential Areas

Industrial Areas

Introduction

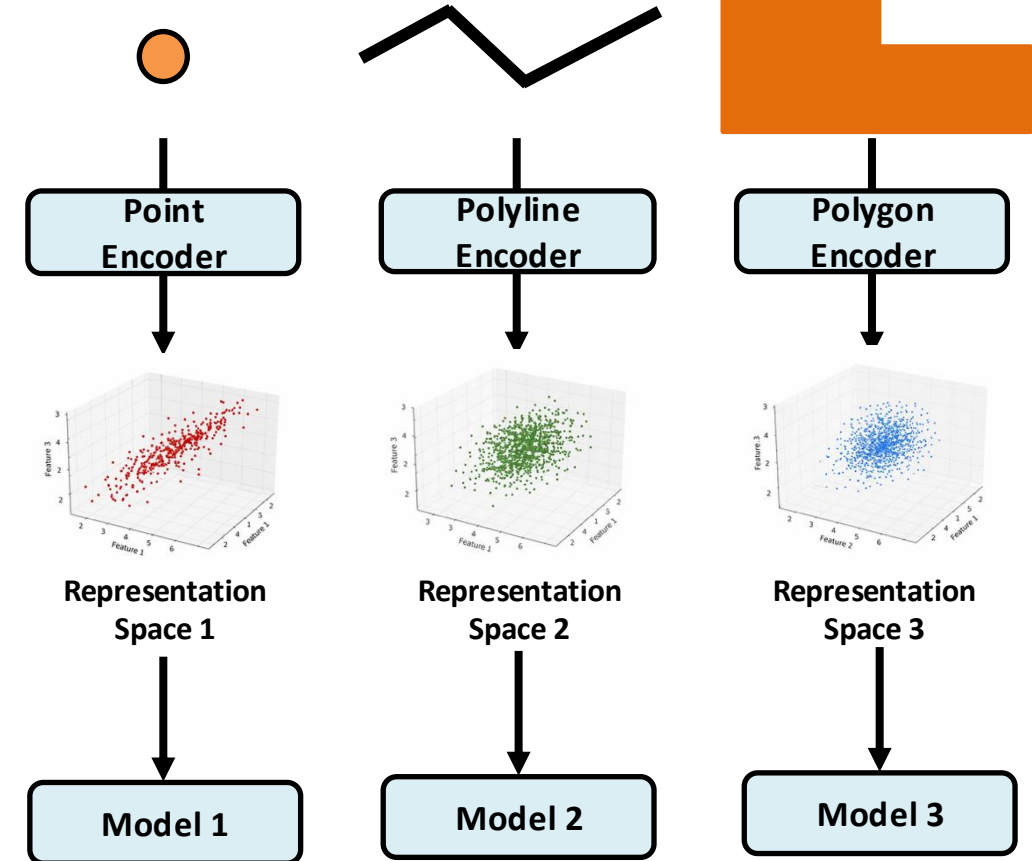


- Geospatial entities include various data structures.
 - Point → POI, Location.
 - Polyline → Road, Trajectory.
 - Polygon → Building Footprint, Boundary.
- **Representation Learning** of geospatial entities is the initial step for GeoAI tasks.
- Existing spatial entity representation methods are largely influenced by node–edge data storage paradigms. Sequential models are used to learn line-like entities, while node-edge models are applied to model polygon entities.

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a = LineString([[0, 0], [1, 0], [1, 1]])
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coords = ((0., 0.), (0., 1.), (1., 1.), (1., 0.), (0., 0.))
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polygon = Polygon(coords)
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Introduction

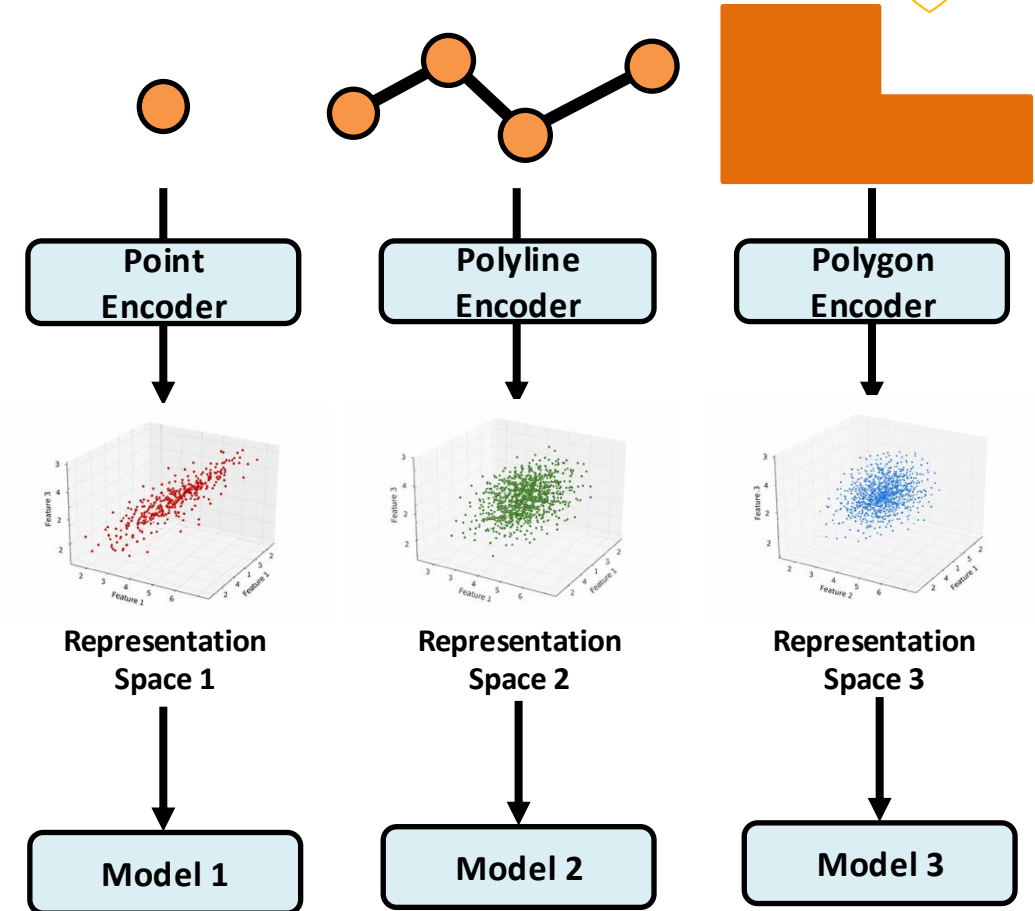


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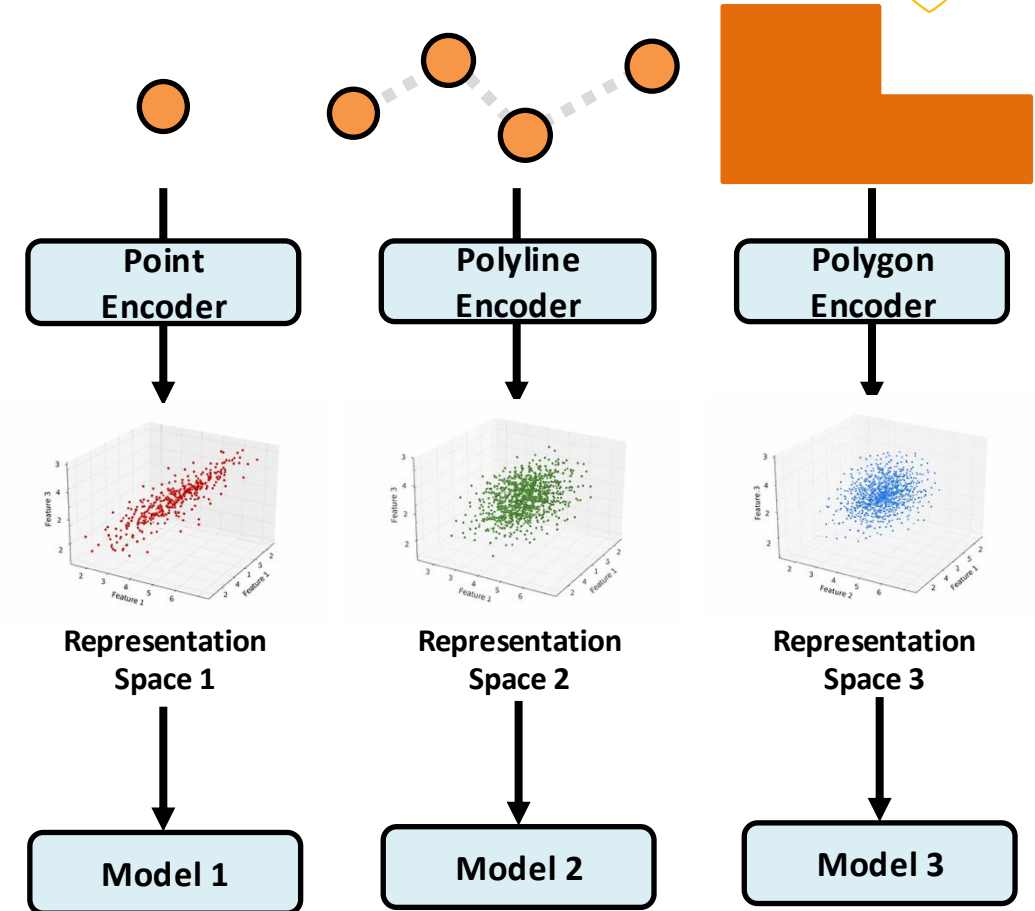


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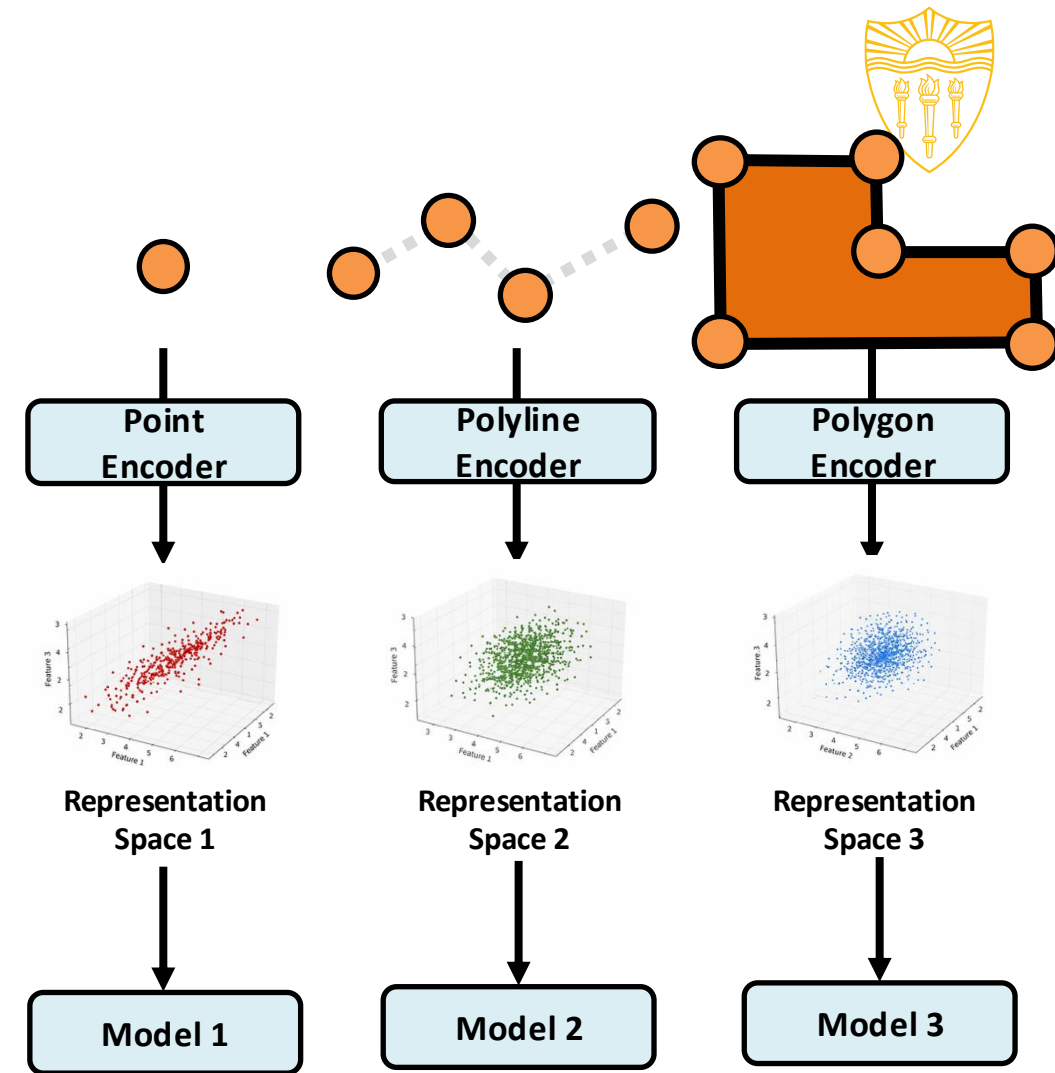
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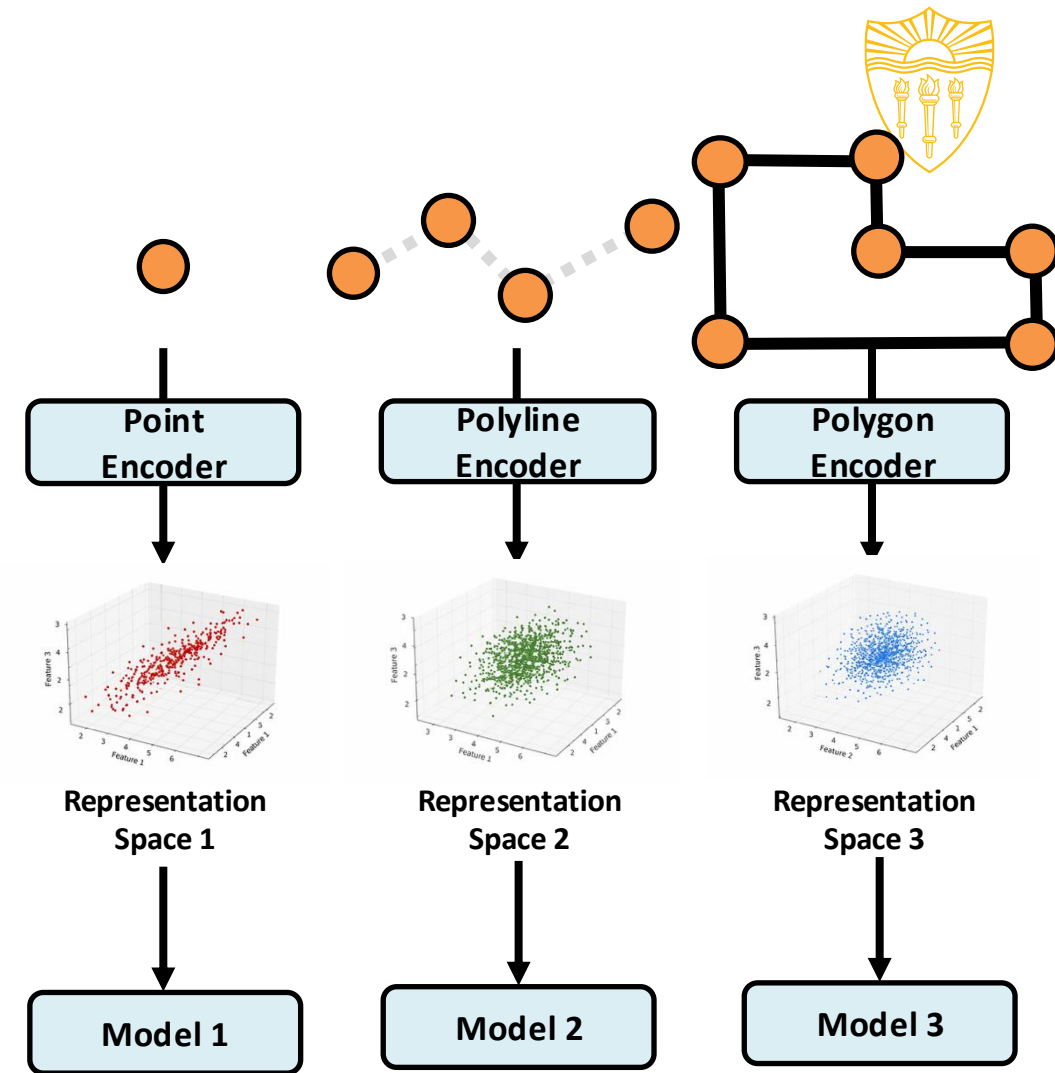
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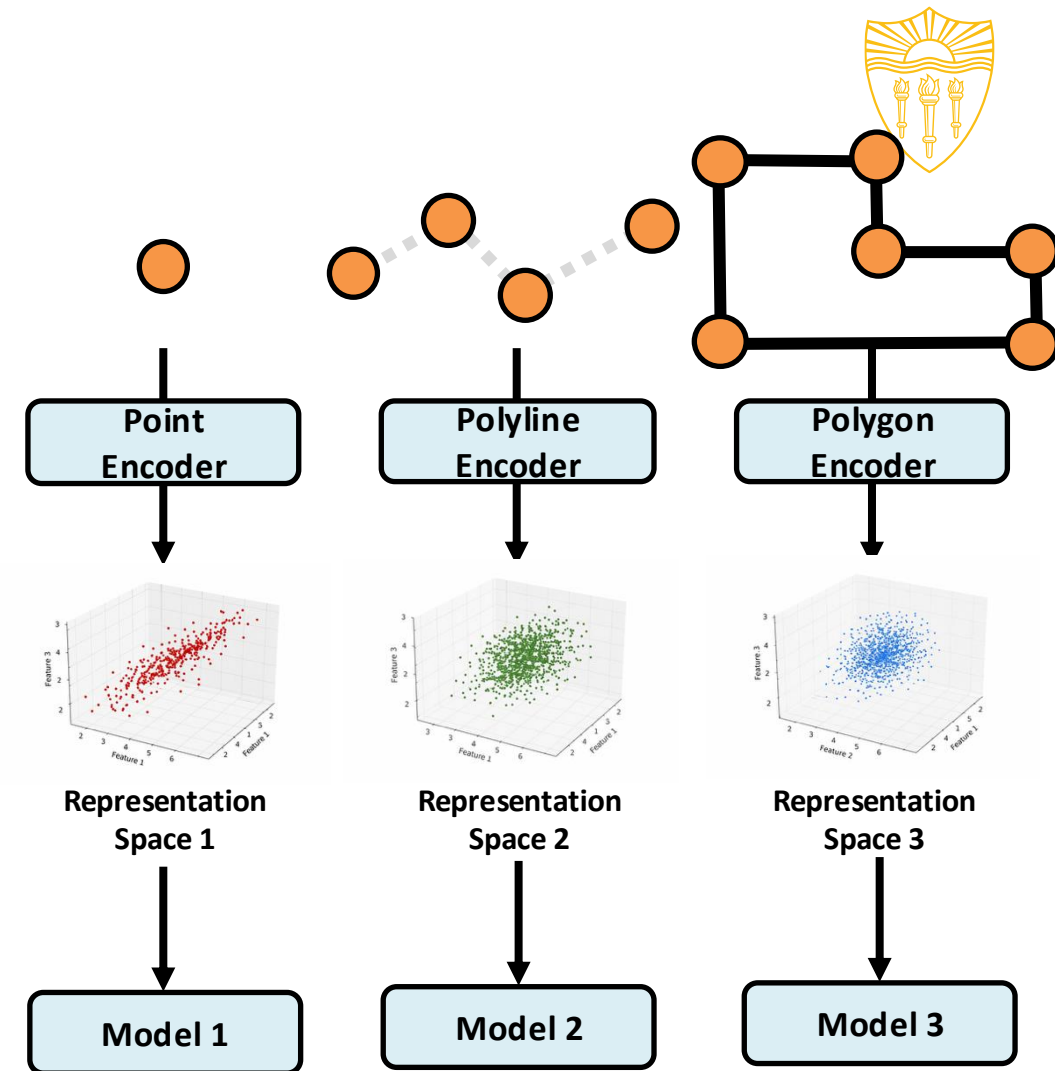
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- Geospatial entities include various data structures.
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- **Representation Learning** of geospatial entities is the initial step for GeoAI tasks.
- **Limitation**: Discrete data structures are not well-suited for geospatial entity representation.

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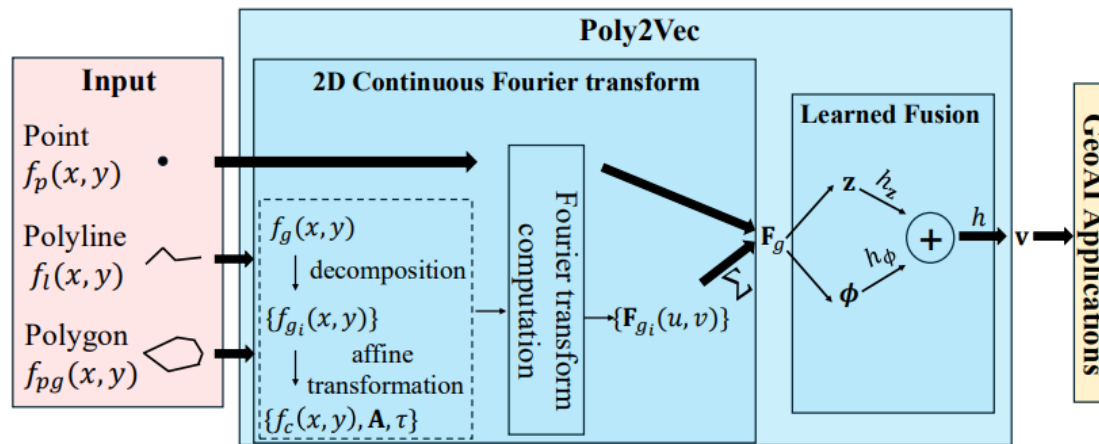


Poly2Vec

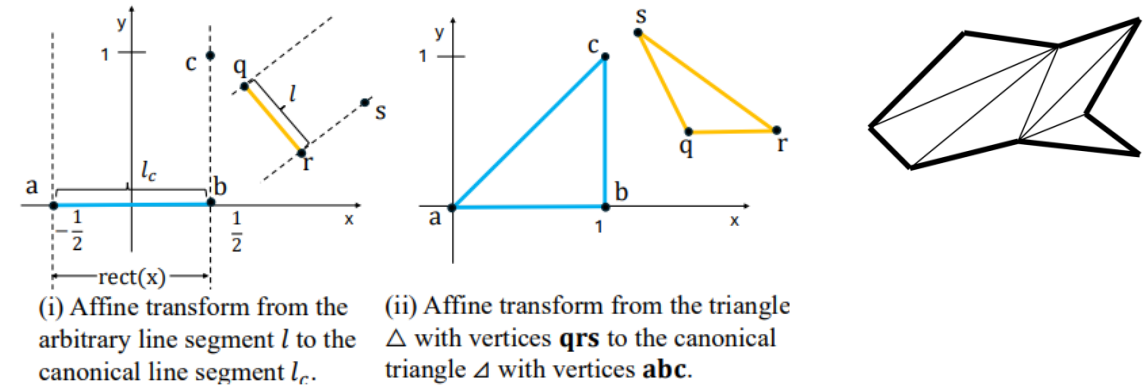
- Poly2Vec:

- Decompose geo-entities to triangles.
- Define a uniform line/triangle, then define the **transform function** from the uniform triangle to triangles.
- Use Fourier transform to representation each **function**. Fourier transform is a linear operation, and addable.

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi\xi x} dx, \quad \forall \xi \in \mathbb{R}. \quad \mathcal{F}\{x(t) + y(t)\} = \mathcal{F}\{x(t)\} + \mathcal{F}\{y(t)\} = X(\omega) + Y(\omega)$$



(a) The workflow of POLY2VEC.



(b) Affine transform arbitrary geometry to its corresponding canonical geometry.

Figure 2: Overview of POLY2VEC.

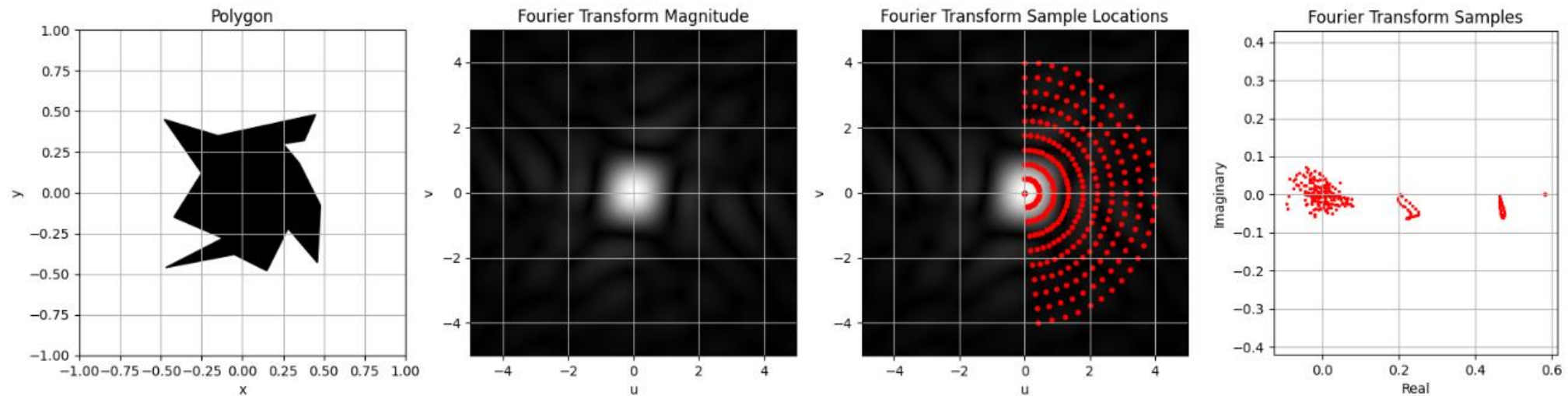


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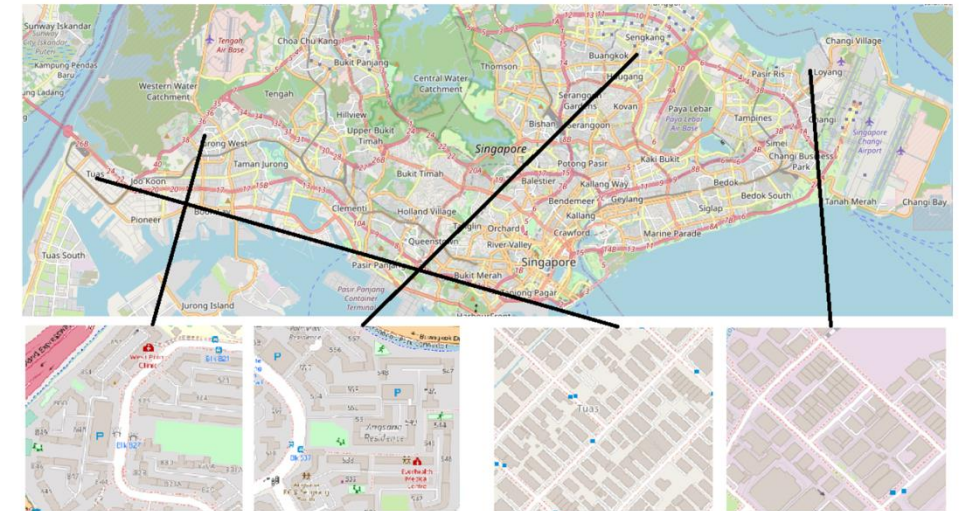
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Table 4: Comparison of methods for Land Use Classification and Population Prediction. **Best** values are highlighted.

Methods	Land Use Classification					
	Singapore			New York		
	L1 ↓	KL ↓	Cosine ↑	L1 ↓	KL ↓	Cosine ↑
RegionDCL	0.498 ± 0.038	0.294 ± 0.047	0.879 ± 0.021	0.418 ± 0.012	0.229 ± 0.013	0.912 ± 0.006
RegionDCL w/o distance-bias	0.558 ± 0.043	0.369 ± 0.067	0.844 ± 0.023	0.439 ± 0.012	0.244 ± 0.012	0.904 ± 0.005
RegionDCL w/ Poly2Vec	0.484 ± 0.021	0.278 ± 0.025	0.881 ± 0.012	0.397 ± 0.010	0.212 ± 0.011	0.923 ± 0.007
Methods	Population Prediction					
	Singapore			New York		
	MAE ↓	RMSE ↓	R ² ↑	MAE ↓	RMSE ↓	R ² ↑
RegionDCL	5807.54 ± 522.74	7942.74 ± 779.44	0.427 ± 0.108	5020.20 ± 216.63	6960.51 ± 282.35	0.575 ± 0.039
RegionDCL w/o distance-bias	6018.94 ± 641.71	8214.58 ± 931.11	0.385 ± 0.087	5293.04 ± 277.31	7348.86 ± 374.62	0.532 ± 0.030
RegionDCL w/ Poly2Vec	4957.58 ± 506.02	6874.47 ± 851.73	0.561 ± 0.117	4602.75 ± 179.66	6393.38 ± 279.70	0.621 ± 0.037



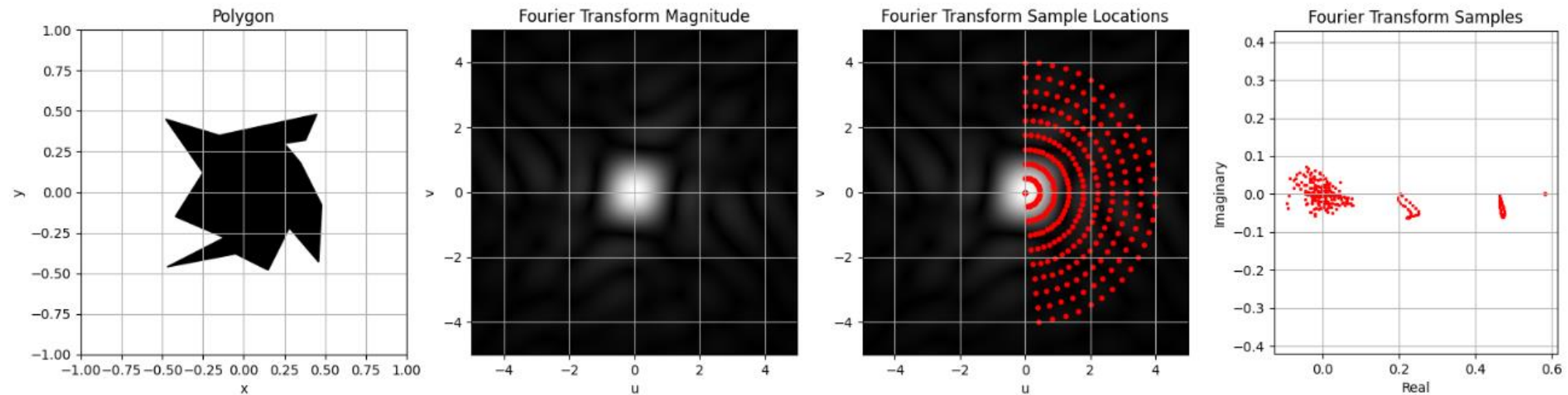
Residential Areas

Industrial Areas

Poly2Vec



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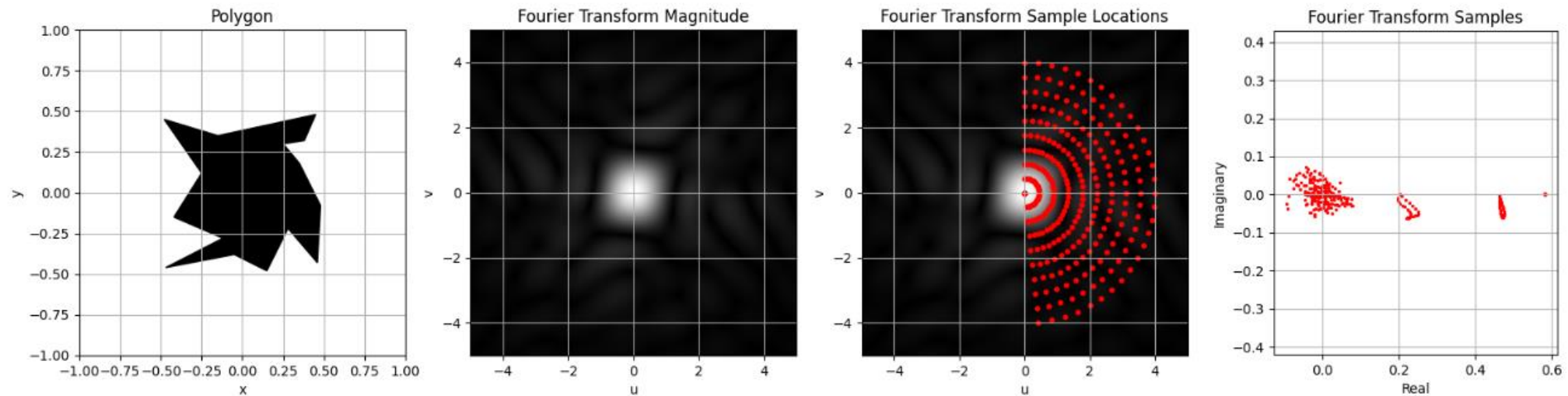


Poly2Vec



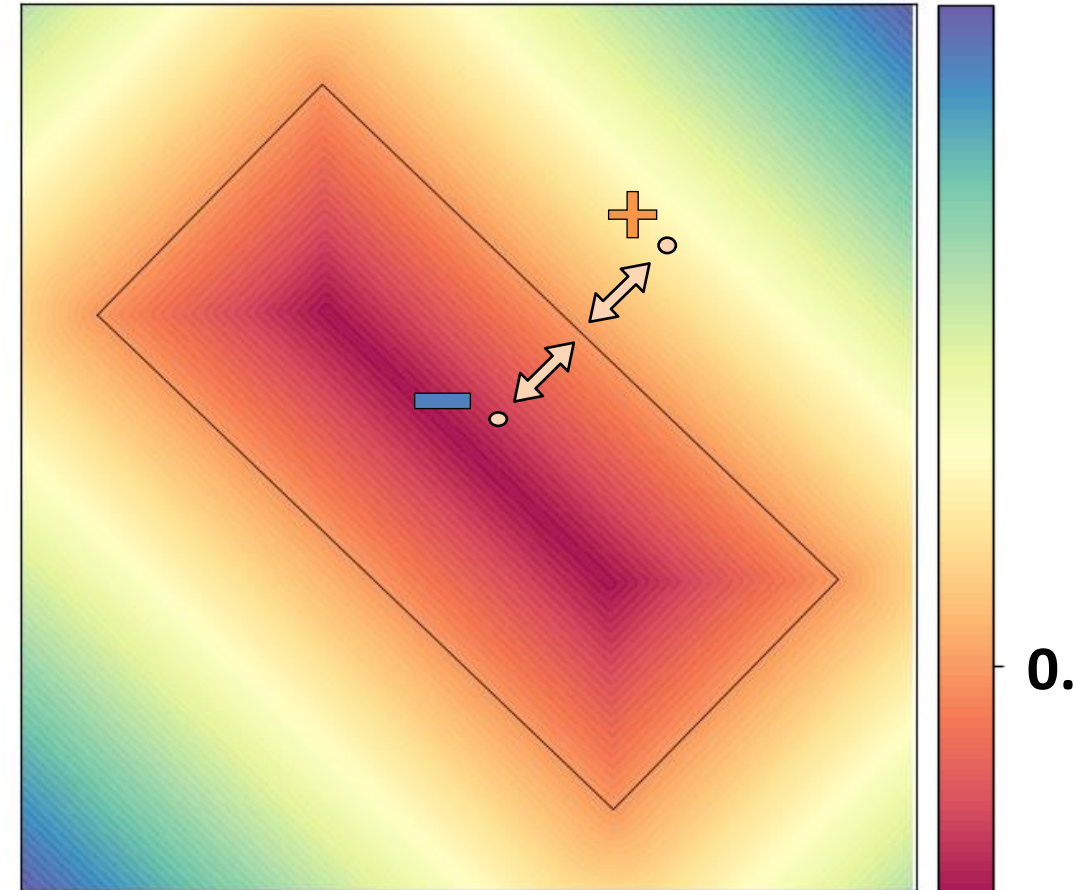
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Limitation: Sampling in the Fourier space is **uninterpretable**.



Geo2Vec: Continuous Representation of Geo-entities

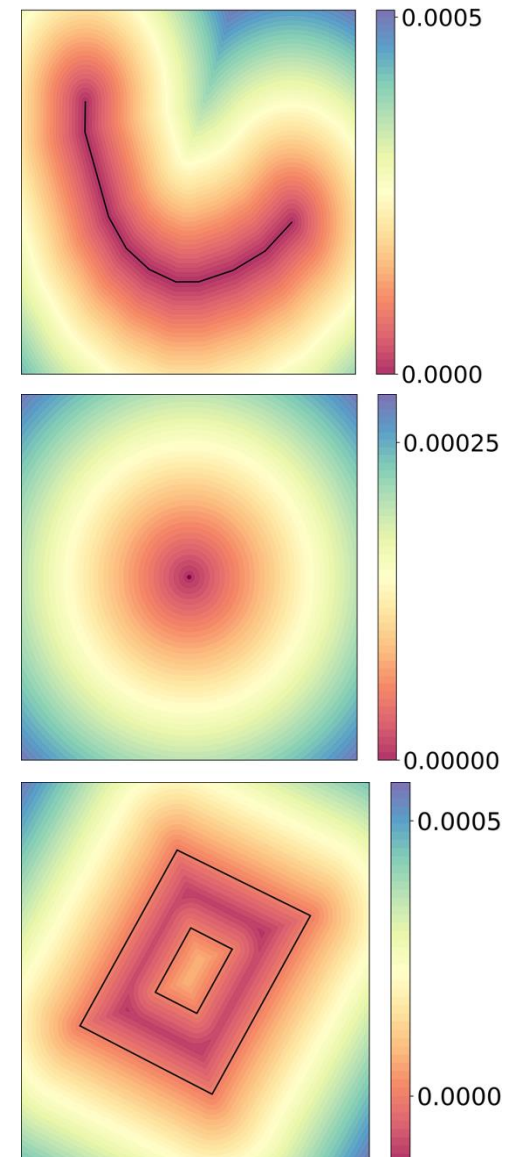
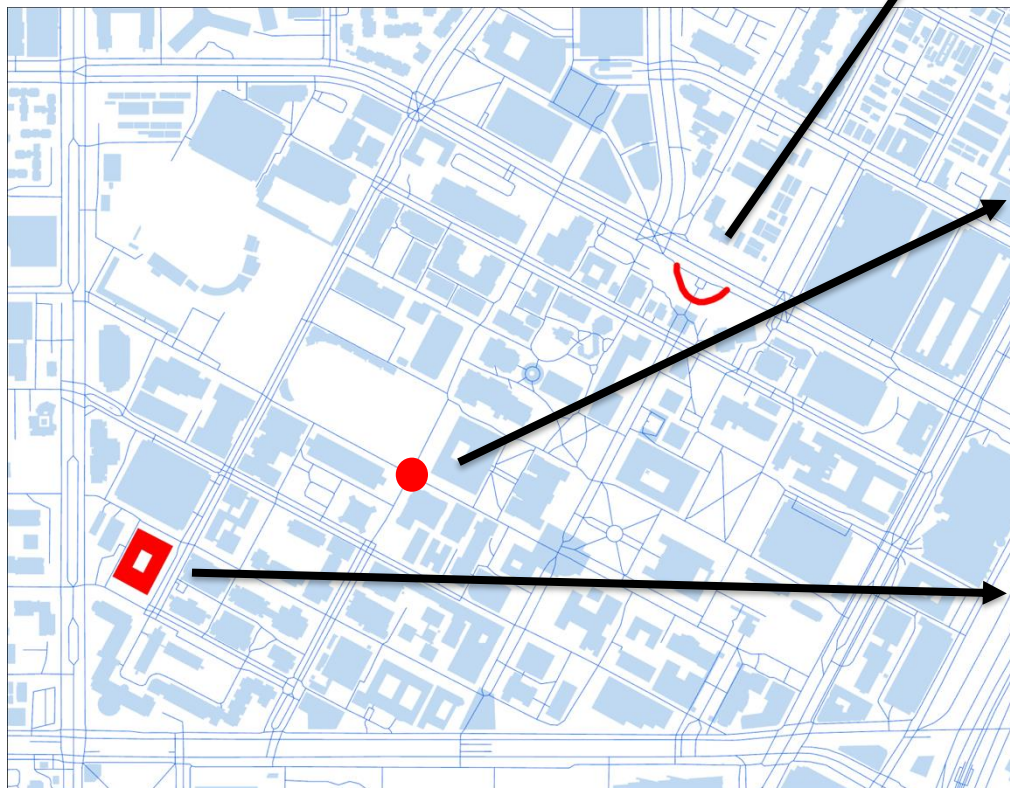
- Using **Signed Distance Field (SDF)** to represent each entity in the continuous space.
- Definition:
Given a geo-entity E , the Signed Distance Field is a scalar field defined over a continuous spatial domain $\Omega_E \subseteq R^2$, in which each point $x \in \Omega_E$ is assigned a scalar value representing its signed distance to E .
- Continuous representation



Unified Representation of Geo-entities

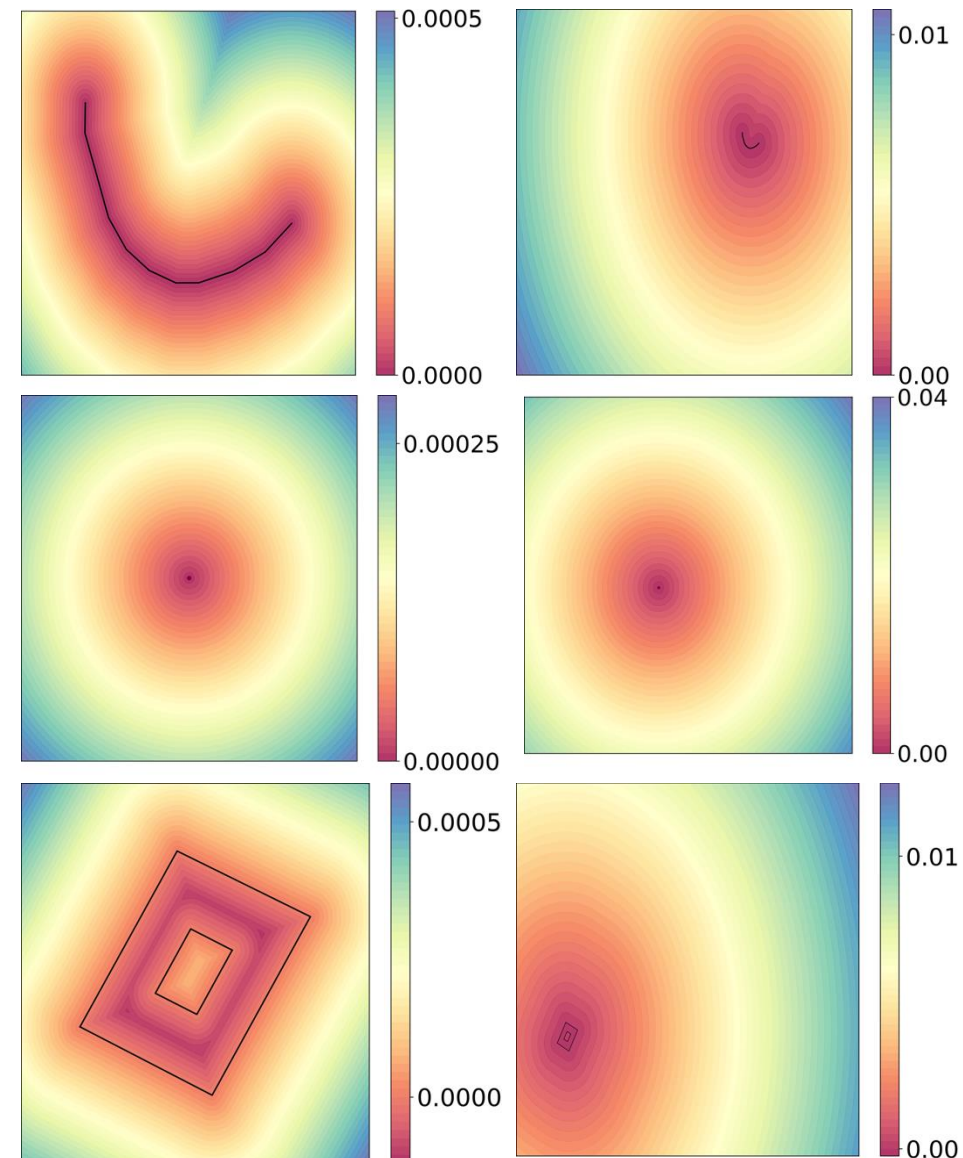
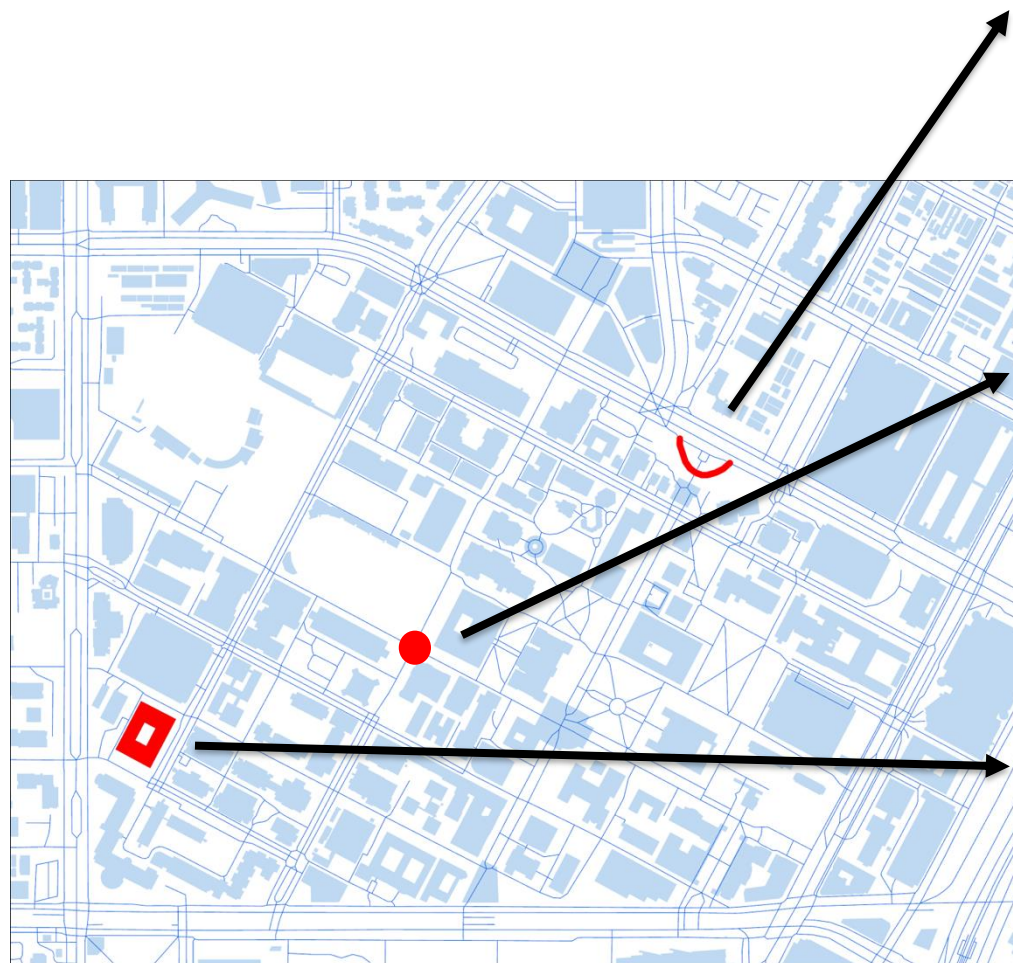


- Point, LineString, MultiLineString
All Positive
- Polygon, MultiPolygon:
Positive outside
Negative Inside

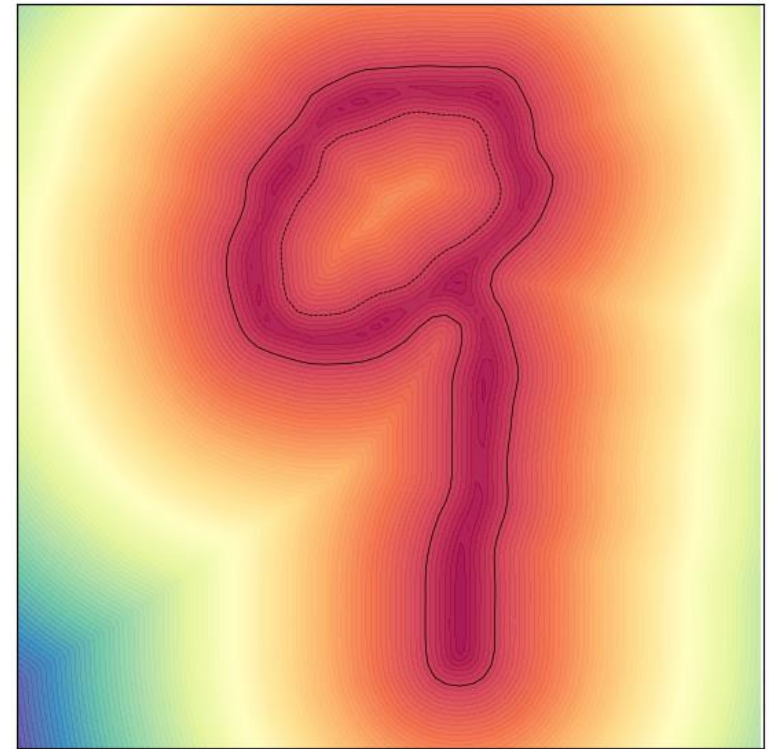
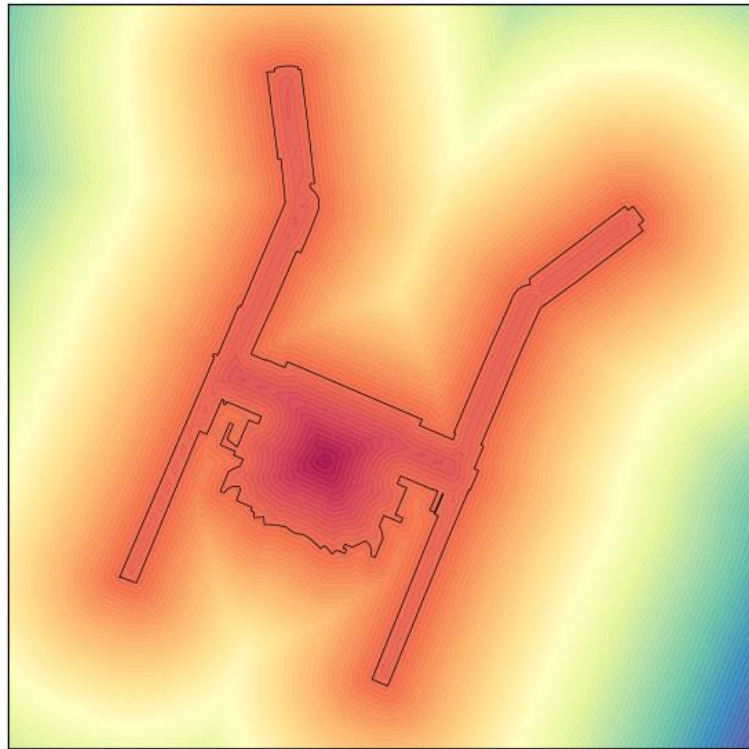
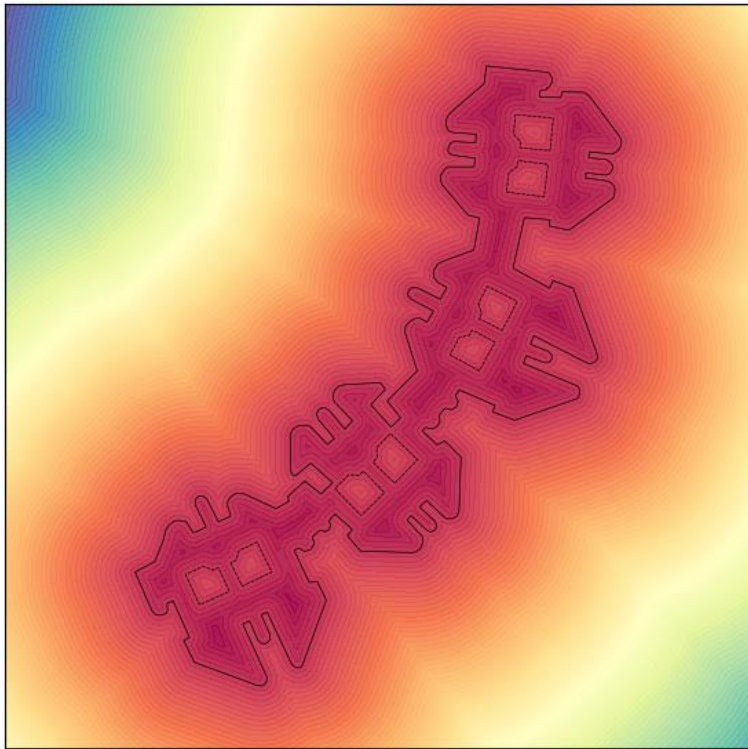
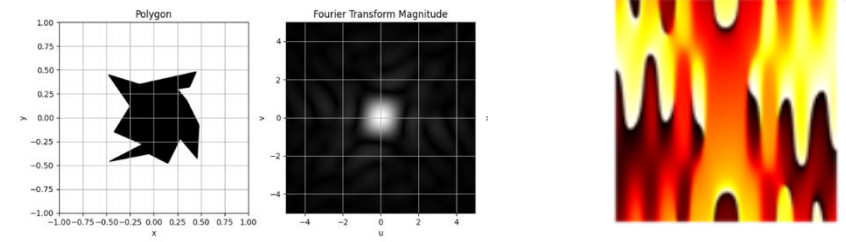


Location-Shape coherent representation

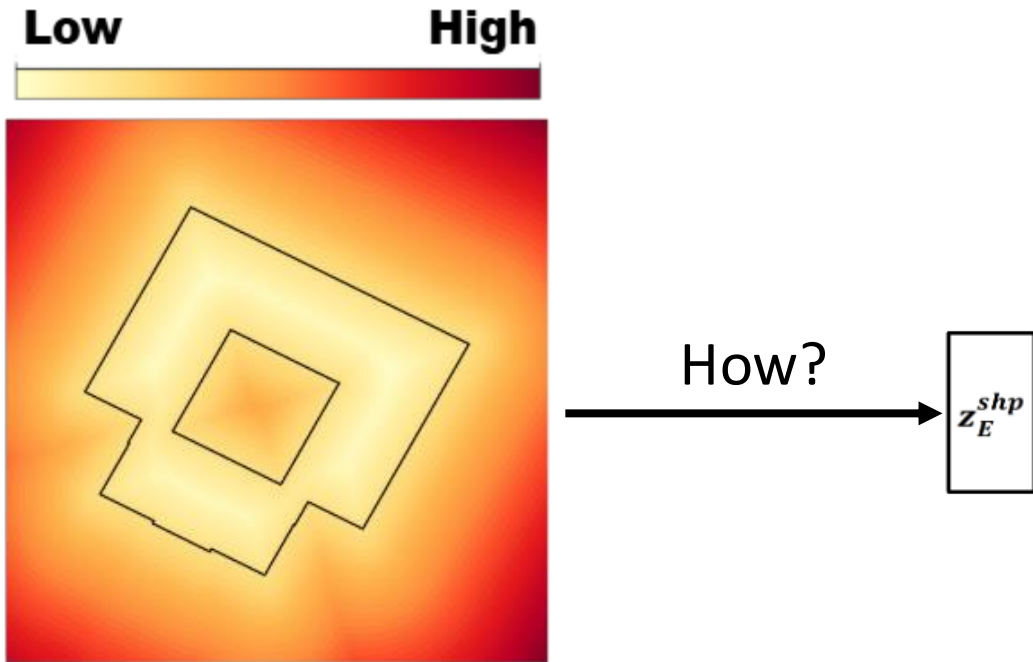
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All Positive
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Positive outside
Negative Inside



Interpretable representation



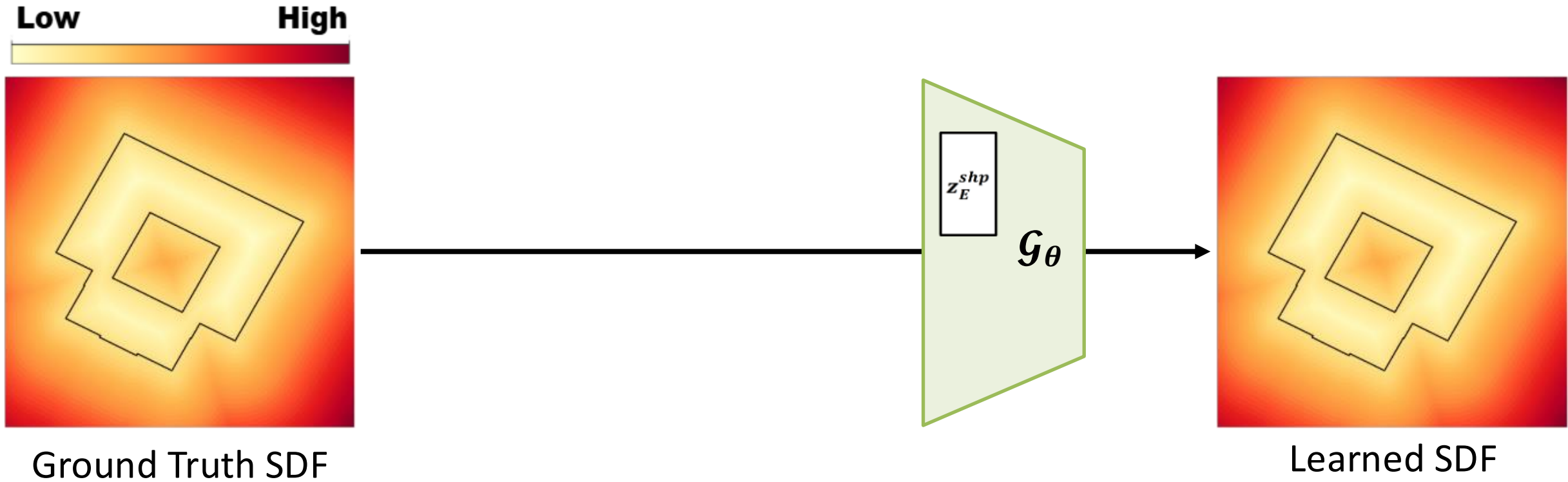
Geo2Vec model





Geo2Vec model

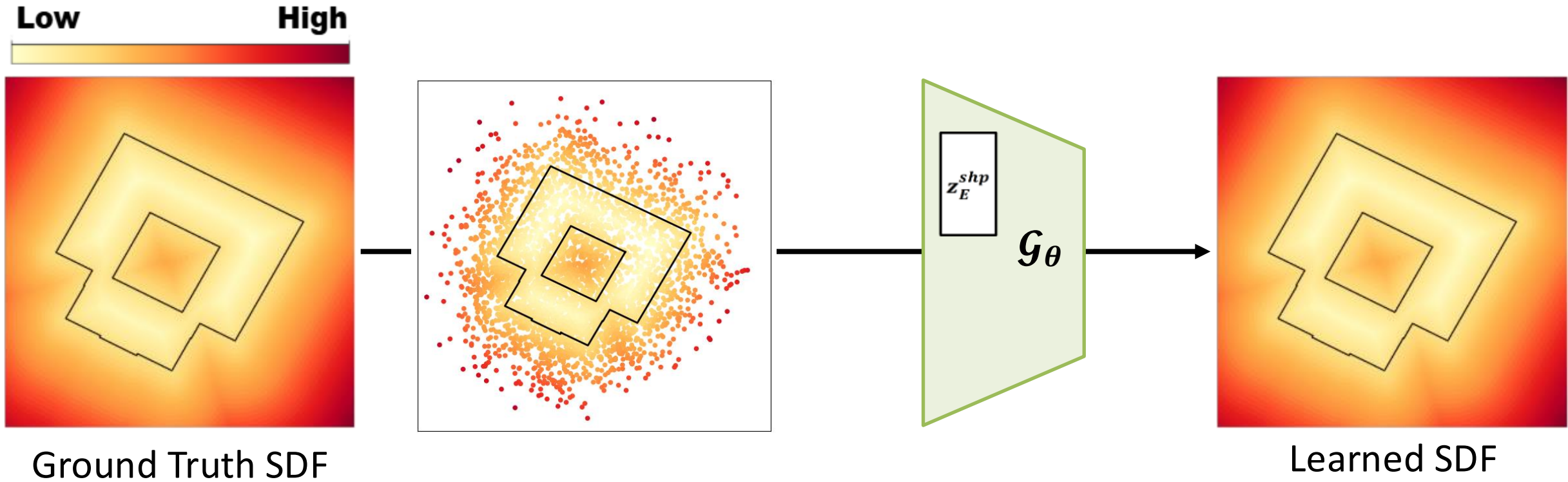
- Approximate the SDF of a geo-entity with a neural network \mathcal{G}_θ .





Geo2Vec model

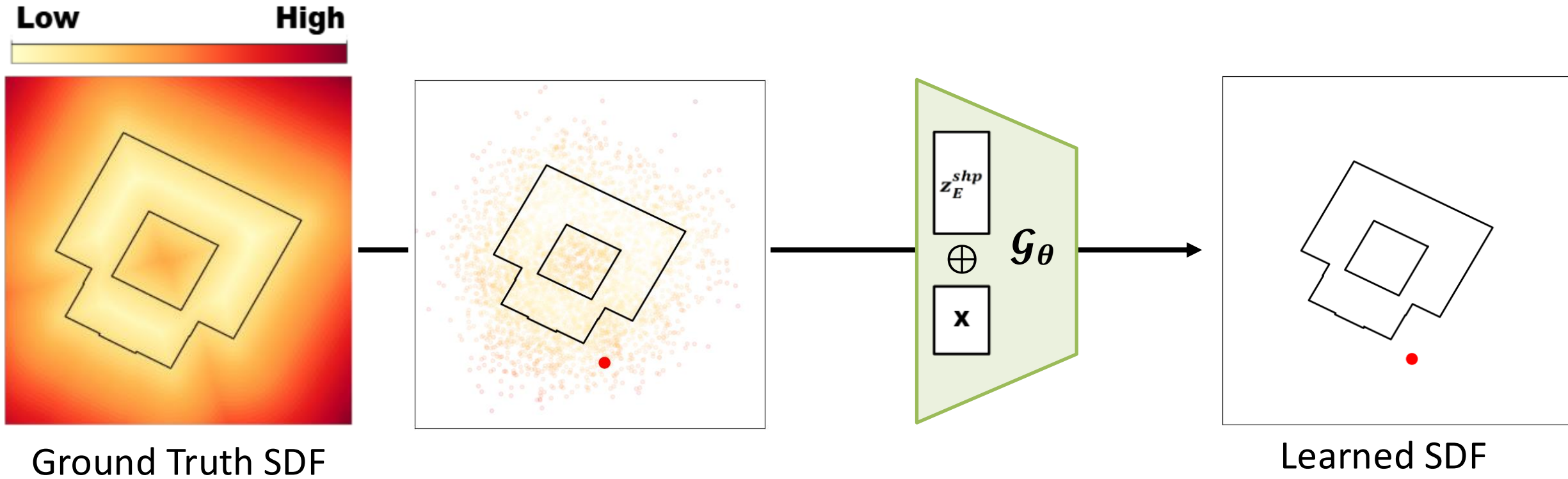
- Approximate the SDF through sampling in the coordinate space.





Geo2Vec model

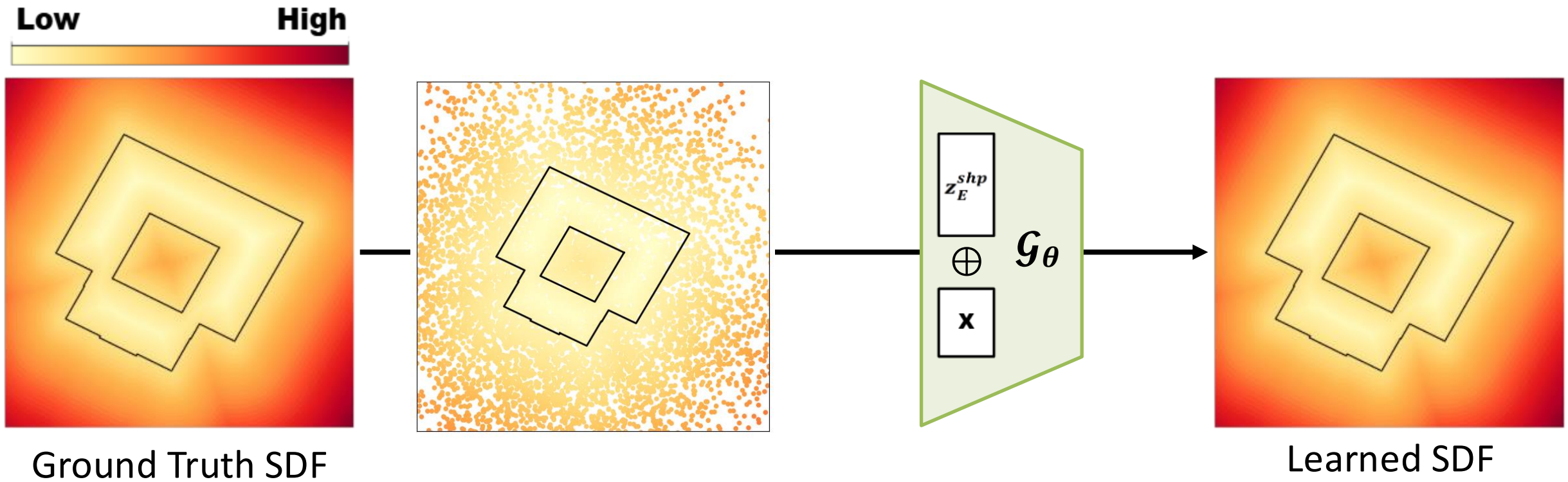
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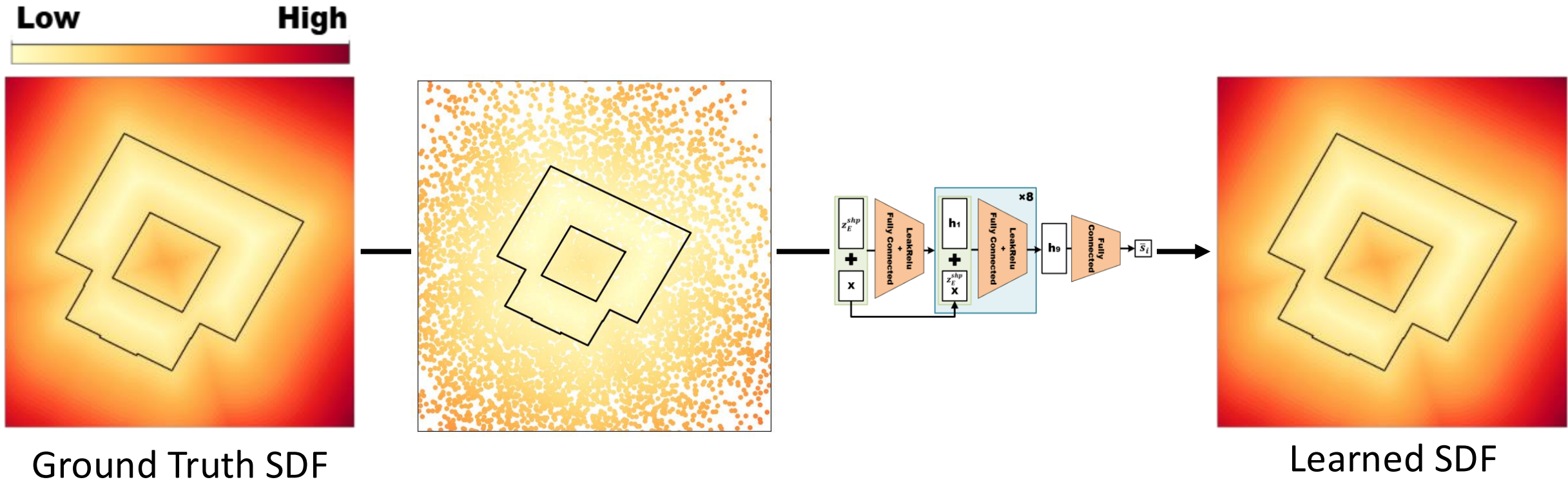
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Geo2Vec model

- Approximate the SDF of a geo-entity with a neural network \mathcal{G}_θ .



Details

- Adaptive & Rotation invariant
Positional Encoder:
Learn shape in detail, learn location more effectively.

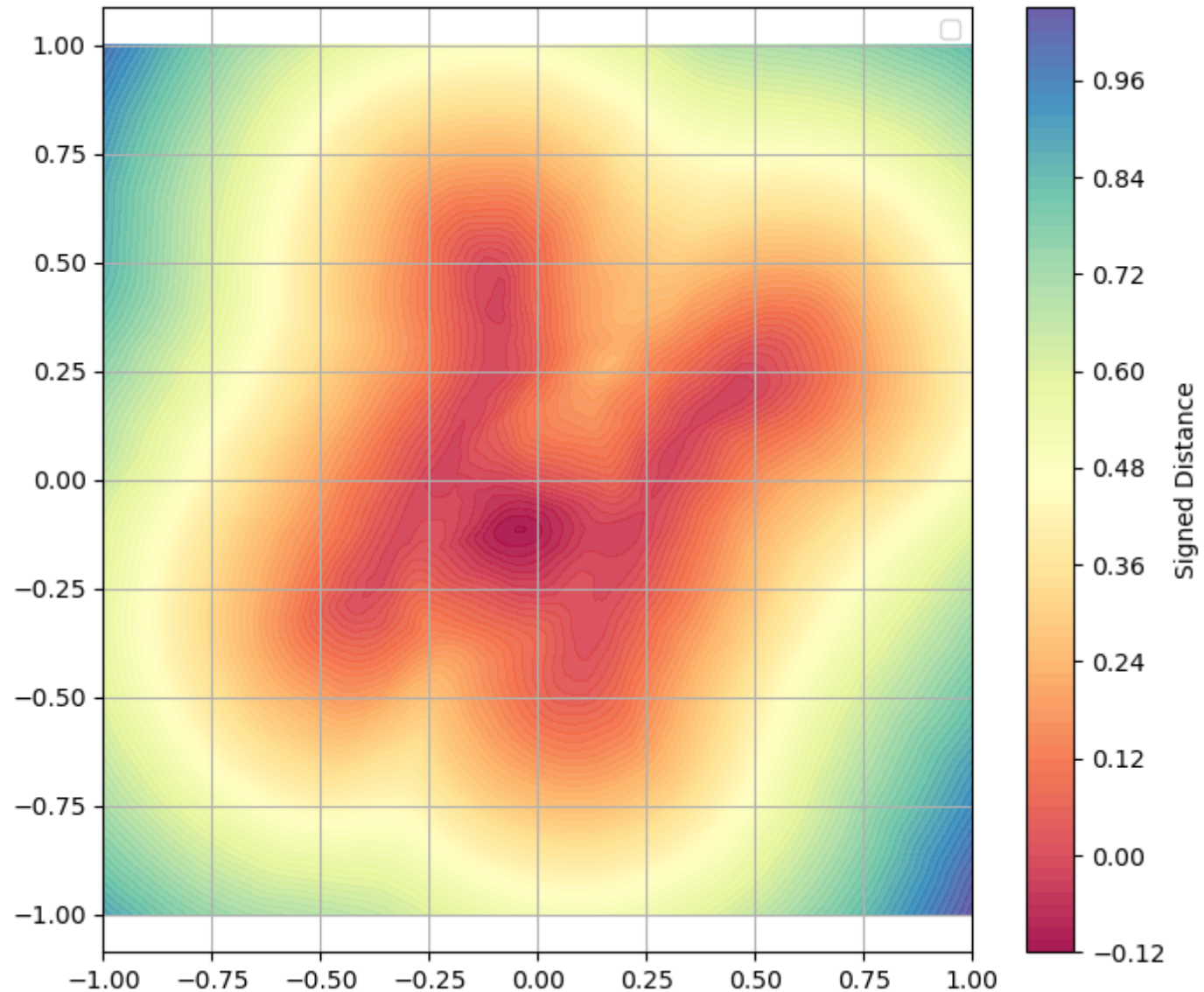
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$$\Delta_x = \max_{E \in G} (E.x) - \min_{E \in G} (E.x), \Delta_y = \max_{E \in G} (E.y) - \min_{E \in G} (E.y),$$

$$\Delta_{\min} = \min(\Delta_x, \Delta_y), \Delta_{\max} = \max(\Delta_x, \Delta_y),$$

$$L_{\max}^{\text{loc}} \leq \log_2 \left(\frac{2}{\Delta_{\min}} \right), \quad L_{\max}^{\text{shp}} \geq \log_2 \left(\frac{2}{\Delta_{\min}} \right),$$

$$L_{\min}^{\text{loc}}, L_{\min}^{\text{shp}} \leq 1 - \log_2 (\Delta_{\max}). \quad (7)$$



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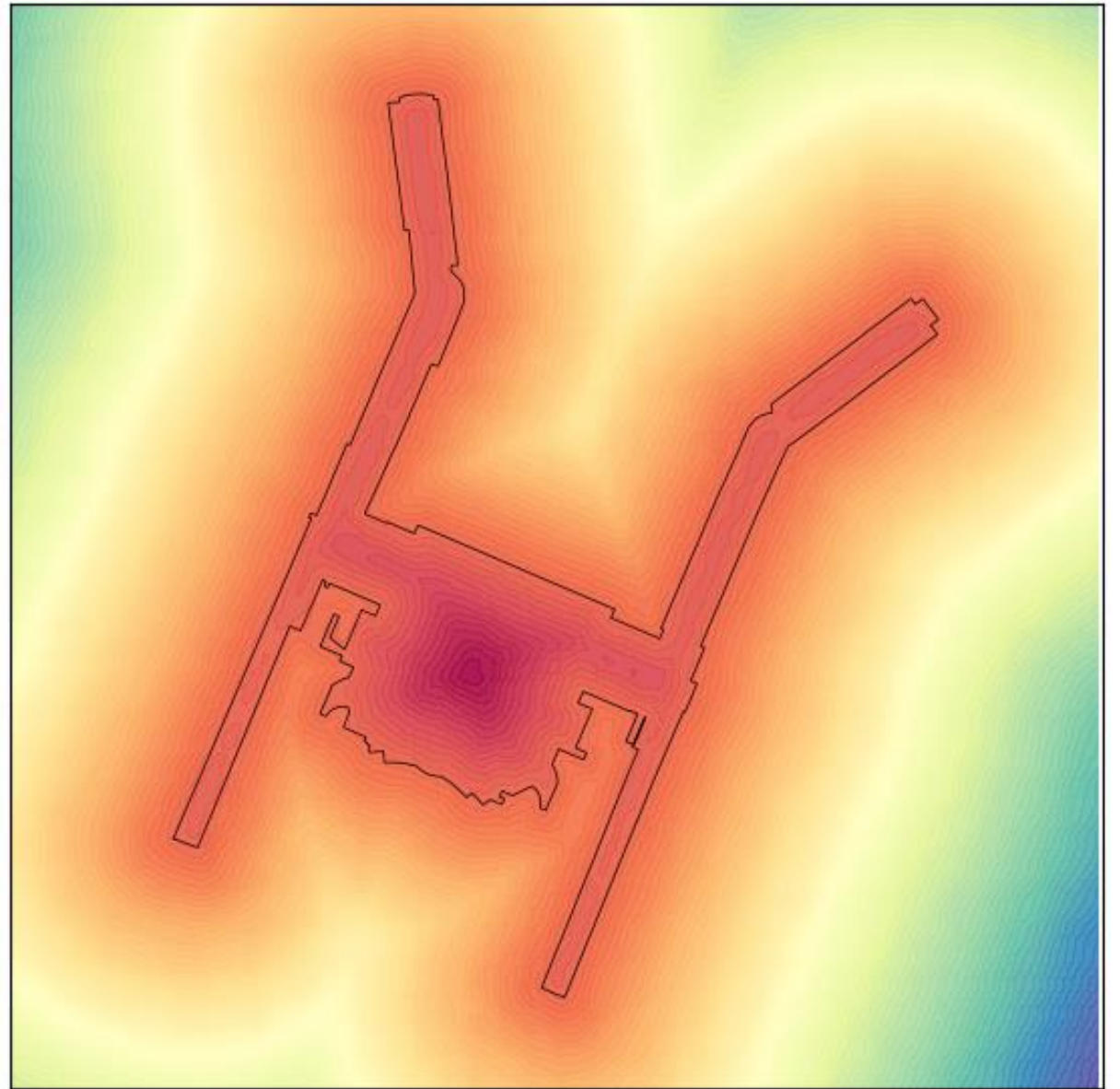
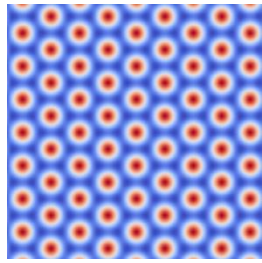
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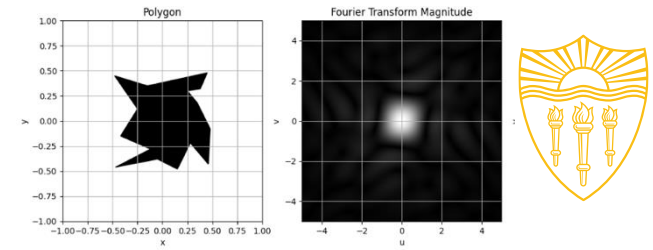
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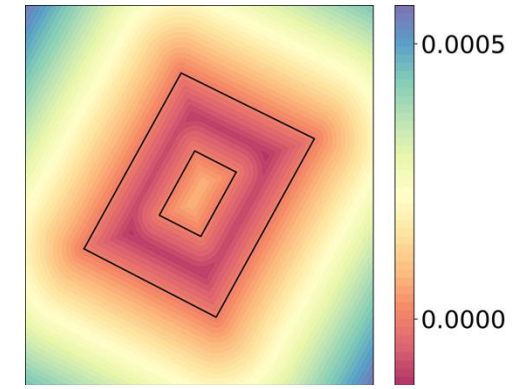
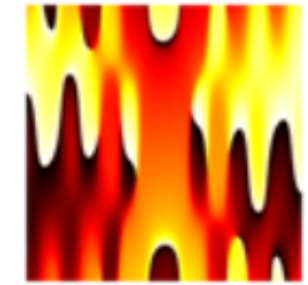
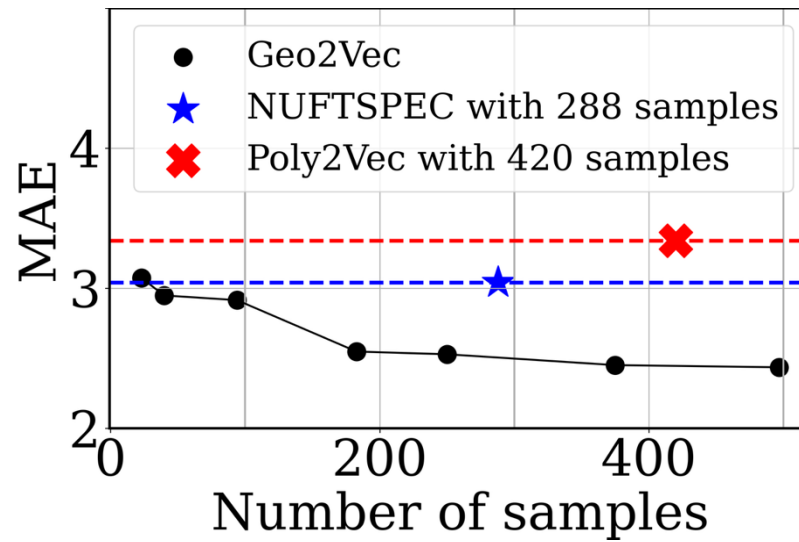
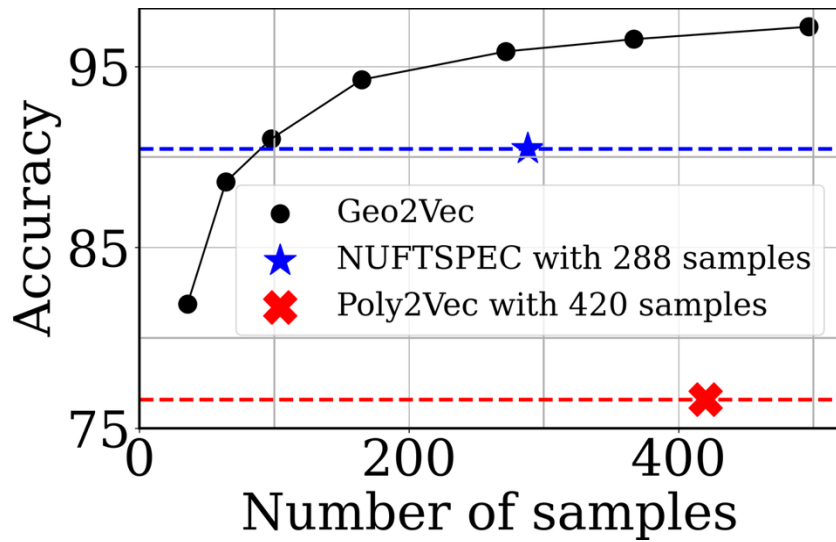
$$L_{\min}^{\text{loc}}, L_{\min}^{\text{shp}} \leq 1 - \log_2 (\Delta_{\max}). \quad (7)$$



Sampling quality



- More efficient sampling in the coordinate space.



Shape Representation



- Up to 61.95% improvement on number of edges prediction of Polygon.
- At least 44.1% improvement in Polyline length representation.

Polyline Representation: MAE performance on Length of Line prediction.

	Singapore	NYC
	Line Length↓	Line Length↓
T2Vec	10.38±0.45	13.20±0.42
T-JEPA	10.25±0.54	12.65±0.36
Poly2Vec	13.55±0.79	21.11±0.48
Geo2Vec	5.75±0.26	7.07±0.16

Polygon Representation: Accuracy on Shape Classification and MAE on the Number of Edges prediction

	Building		MNIST		Singapore	NYC
	Shape↑	Edge↓	Shape↑	Edge↓	Edge↓	Edge↓
PolyGNN	87.84±0.005	–	7.77±0.013	–	–	–
NUFTSPEC	90.46±0.730	3.04±0.125	96.90±0.116	16.76±0.588	3.66±0.023	1.45±0.020
Poly2Vec	76.59±1.403	3.34±0.127	92.52±0.265	29.29±0.807	3.68±0.109	1.21±0.045
Geo2Vec	97.34±0.310	2.22±0.050	97.58±0.097	9.45±0.124	1.40±0.011	0.72±0.002

Location Representation



- Up to 54.3% improvement on Topological Relationship Classification.
- More **uniformed** performance when predicting the topology and distance between **different types of entities**.

Overall model performance on distance estimation.

	Building	MNIST	Singapore			NYC		
	Pg-Pg↓	Pg-Pg↓	Pt-Pg↓	Pl-Pg↓	Pg-Pg↓	Pt-Pg↓	Pl-Pg↓	Pg-Pg↓
TILE	217.1±1.6	223.8±1.0	99.9±1.7	115.5±1.5	114.3±1.4	127.6±0.7	154.3±2.1	167.4±2.0
THEORY	7.3±4.3	34.2±1.0	24.3±0.9	25.0±0.4	25.0±1.1	26.2±1.1	26.6±0.5	27.4±0.7
Poly2Vec	13.1±1.0	21.0±0.4	15.9±0.6	19.9±1.7	22.0±0.6	28.7±1.4	28.5±0.4	52.7±0.8
Geo2Vec	6.4±0.9	13.0±0.8	5.4±0.5	5.0±0.1	5.5±0.4	10.2±0.1	13.0±0.9	12.9±0.6

Model accuracy on Topological Relationship Classification.

	Singapore					NYC				
	Pt-Pl↑	Pt-Pg↑	Pl-Pl↑	Pl-Pg↑	Pg-Pg↑	Pt-Pl↑	Pt-Pg↑	Pl-Pl↑	Pl-Pg↑	Pg-Pg↑
NUFTSPEC	-	-	-	-	60.2±0.9	-	-	-	-	58.5±0.8
T2VEC	-	-	72.8±2.3	-	-	-	-	80.7±12.1	-	-
T-JEPA	-	-	75.4±1.8	-	-	-	-	79.8±8.6	-	-
DIRECT	82.3±1.3	84.3±0.5	73.3±0.7	36.8±1.0	35.7±1.8	84.6±1.1	90.9±1.8	74.5±0.8	49.5±0.9	44.6±2.3
TILE	79.0±2.1	70.0±1.0	50.5±0.5	45.9±1.3	41.1±1.3	65.9±1.3	78.3±0.7	50.2±0.9	49.4±3.8	40.5±0.5
WRAP	88.6±0.3	88.0±0.8	71.6±1.1	47.6±1.0	47.6±1.0	88.6±0.6	88.0±1.7	73.3±0.9	55.0±1.1	38.1±0.7
GRID	84.6±0.4	84.4±0.4	69.7±3.1	45.8±0.4	45.8±0.4	82.2±3.9	89.1±0.4	73.9±0.9	51.6±0.8	38.1±3.1
THEORY	89.2±0.3	90.0±0.5	71.9±0.8	45.0±1.0	45.0±1.0	89.7±0.8	90.9±0.8	73.4±0.8	59.1±0.6	45.5±4.1
Poly2Vec	95.5±0.7	94.9±0.2	81.2±1.0	50.9±0.8	70.2±0.6	95.3±0.3	98.0±0.2	83.0±0.4	64.1±6.2	68.4±0.8
Geo2Vec	98.5±0.3	96.1±0.2	96.4±0.5	61.2±0.4	75.6±0.4	98.7±0.4	99.1±0.3	98.9±0.3	67.5±0.8	70.0±0.4

Conclusions



- Spatial Representation Learning is the first step for building a Geospatial AI model.
- SRL introduces geospatial inductive bias, making the model generalizable to unseen regions.
- Poly2Vec and Geo2Vec are unified representation methods for geospatial entities.

