Spatial Index Structures

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Outline

- Grid File
- Z-ordering
- Hilbert Curve
- Quad Tree
  - PM
  - PR
- R Tree (next session)
  - R* Tree
  - R+ Tree

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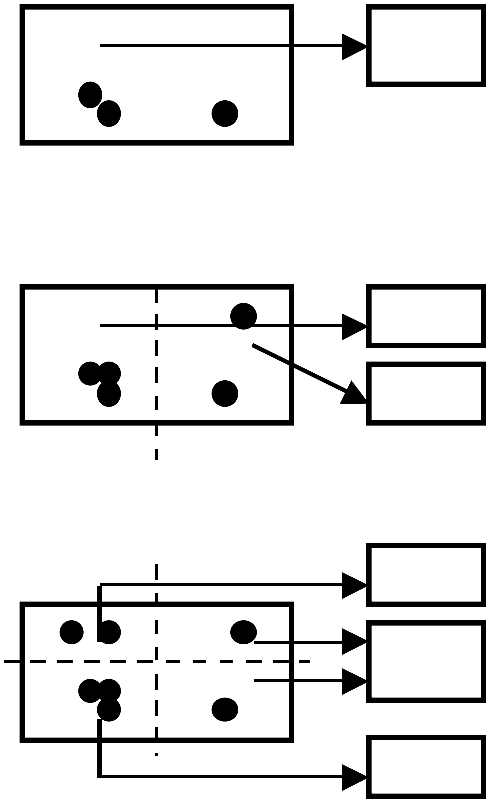
Grid File

- Hashing methods for multidimensional points (extension of Extensible hashing)
- Idea: Use a grid to partition the space→ each cell is associated with one page
- Two disk access principle (exact match)

Grid File



- Start with one bucket for the whole space.
- Select dividers along each dimension. Partition space into cells
- Dividers cut all the way.

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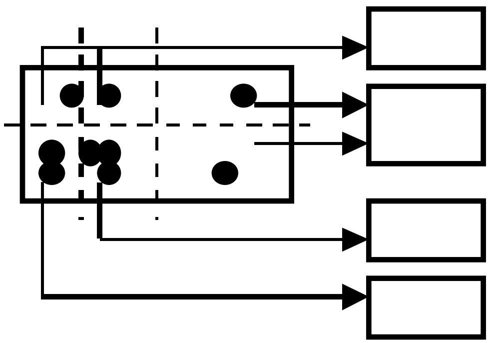
Grid File



- Each cell corresponds to 1 disk page.
- Many cells can point to the same page.
- Cell directory potentially exponential in the number of dimensions

Grid File Implementation

- Dynamic structure using a grid directory
  - Grid array: a 2 dimensional array with pointers to buckets (this array can be large, disk resident) G(0,…, nx-1, 0, …, ny-1)
  - Linear scales: Two 1 dimensional arrays that used to access the grid array (main memory) X(0, …, nx-1), Y(0, …, ny-1)

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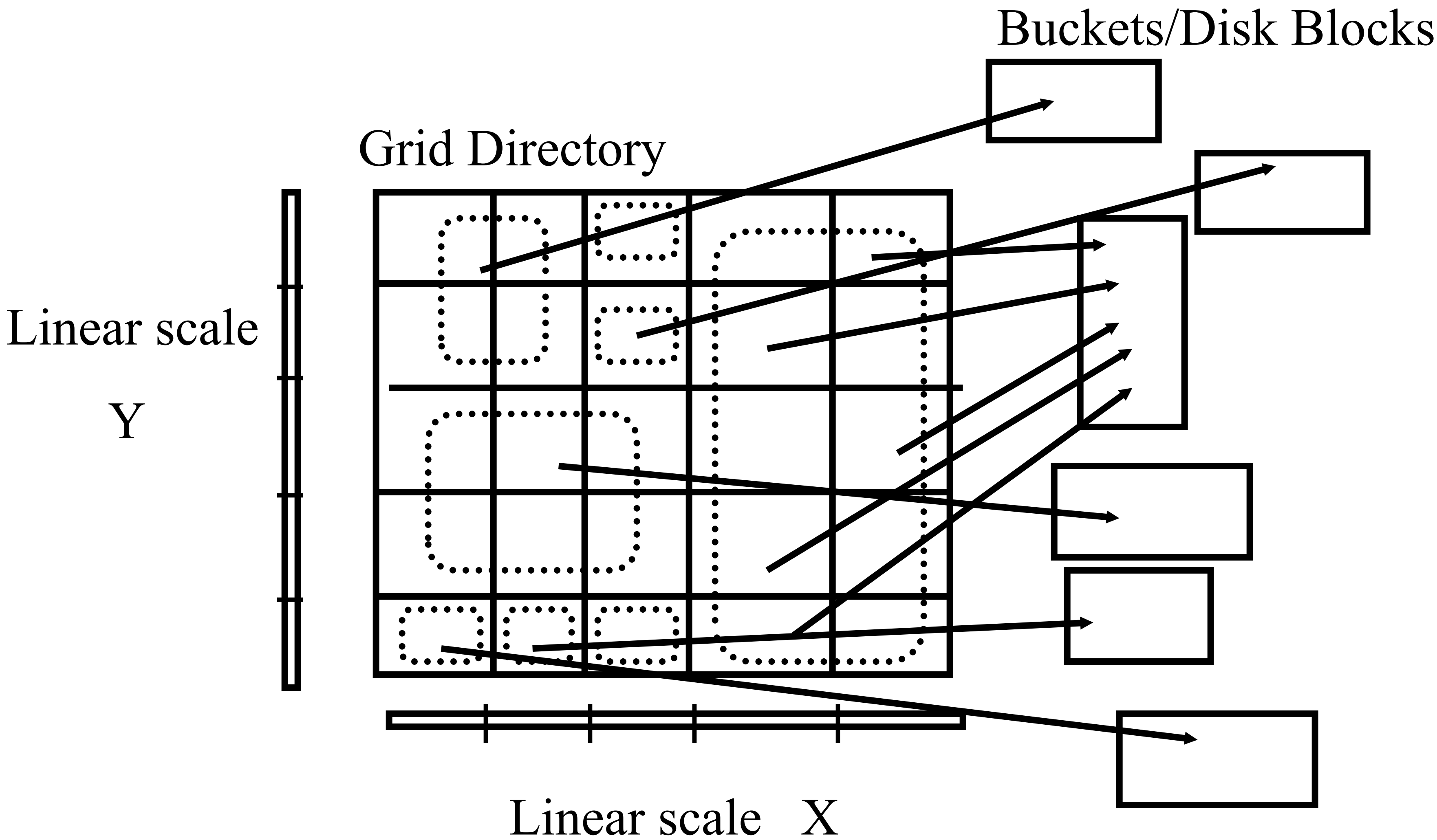
Example



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Grid File Search

- Exact Match Search: at most 2 I/Os assuming linear scales fit in memory.
  - First use liner scales to determine the index into the cell directory
  - access the cell directory to retrieve the bucket address (may cause 1 I/O if cell directory does not fit in memory)
  - access the appropriate bucket (1 I/O)
  - E.g., X=(0, 1000, 1500, 1750, 1875, 2000) ; Y=(a, f, k, p, z)

    --- search for [1980,w]
- Range Queries:
  - use linear scales to determine the index into the cell directory.
  - Access the cell directory to retrieve the bucket addresses of buckets to visit.
  - Access the buckets.

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Grid File Insertions

- Determine the bucket into which insertion must occur.
- If space in bucket, insert.
- Else, split bucket
  - how to choose a good dimension to split?
  - ans: create convex regions for buckets.
- If bucket split causes a cell directory to split do so and adjust linear scales.
- insertion of these new entries potentially requires a complete reorganization of the cell directory--- expensive!!!

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Grid File Deletions

- Deletions may decrease the space utilization. Merge buckets
- We need to decide which cells to merge and a merging threshold
- Buddy system and neighbor system
  - A bucket can merge with only one *buddy* in each dimension
  - Merge adjacent regions if the result is a rectangle

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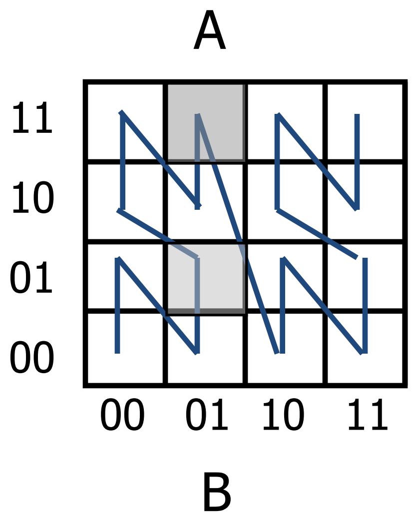
Z-ordering

- Basic assumption: Finite precision in the representation of each coordinate, K bits (2^K values)
- The address space is a square (image) and represented as a 2^K x 2^K array
- Each element is called a pixel

Z-ordering

- Impose a linear ordering on the pixels of the image → 1 dimensional problem



$Z_A = \text{shuffle}(x_A, y_A) = \text{shuffle}(\text{“01”}, \text{“11”})$

$= 0111 = (7)_{10}$

$Z_B = \text{shuffle}(\text{“01”}, \text{“01”}) = 0011$

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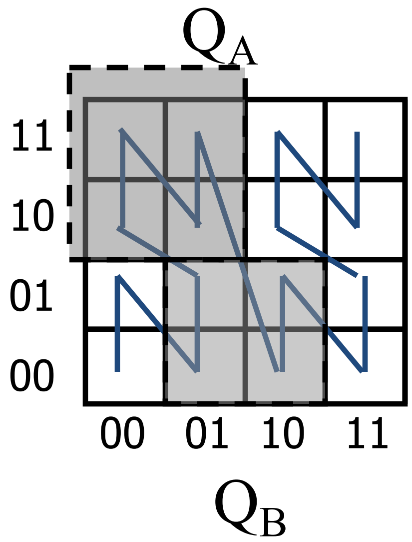
Z-ordering

- Given a point (x, y) and the precision K find the pixel for the point and then compute the z-value
- Given a set of points, use a B+-tree to index the z-values
- A range (rectangular) query in 2-d is mapped to a set of ranges in 1-d

Queries

- Find the z-values that contained in the query and then the ranges



Q_A → range [4, 7]

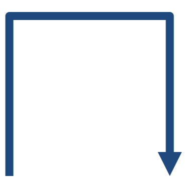
Q_B → ranges [2,3] and [8,9]

Hilbert Curve

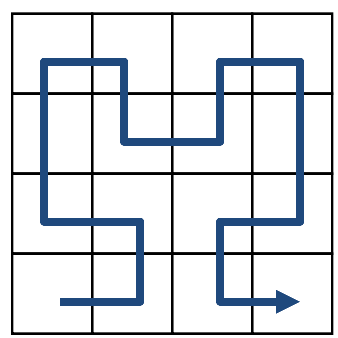
- We want points that are close in 2d to be close in the 1d
- Note that in 2d there are 4 neighbors for each point where in 1d only 2.
- Z-curve has some “jumps” that we would like to avoid
- Hilbert curve avoids the jumps : recursive definition

Hilbert Curve- example

- It has been shown that in general Hilbert is better than the other space filling curves for retrieval *
- H_i (order-i) Hilbert curve for $2^i x 2^i$ array

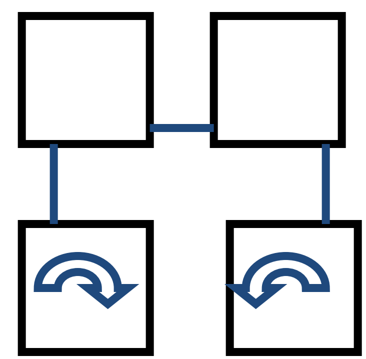


H_1



H_2

…



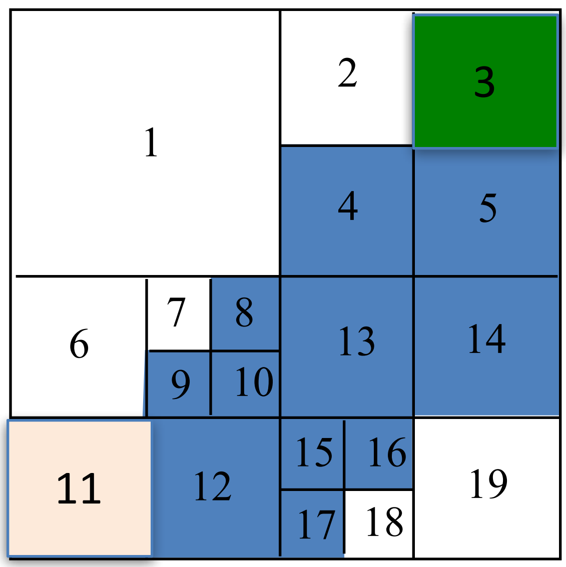
$H_{(n+1)}$

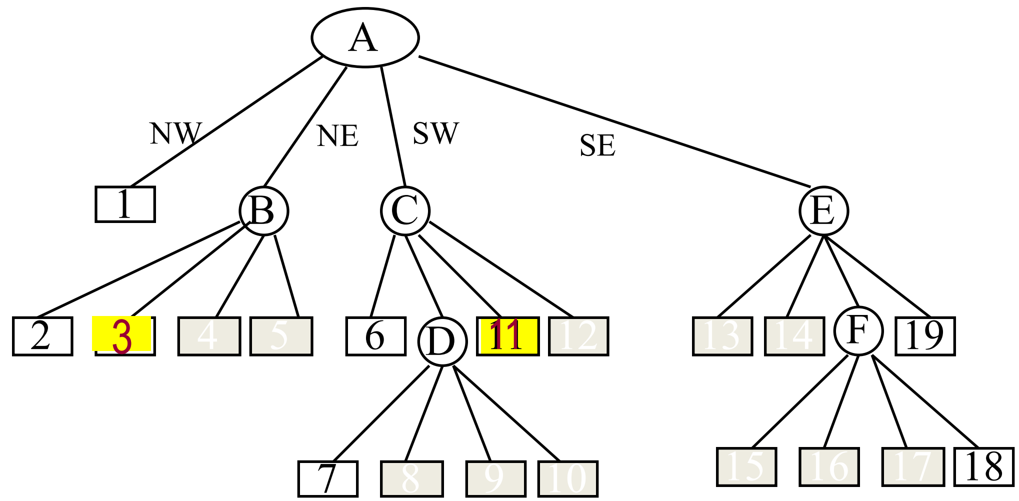
* H. V. Jagadish: Linear Clustering of Objects with Multiple Atributes. ACM SIGMOD Conference 1990: 332-342

Quad Trees

- Region Quadtree
  - The blocks are required to be disjoint
  - Have standard sizes (squares whose sides are power of two)
  - At standard locations
  - Based on successive subdivision of image array into four equal-size quadrants
  - If the region does not cover the entire array, subdivide into quadrants, sub-quadrants, etc.
  - A variable resolution data structure

Example of Region Quadtree





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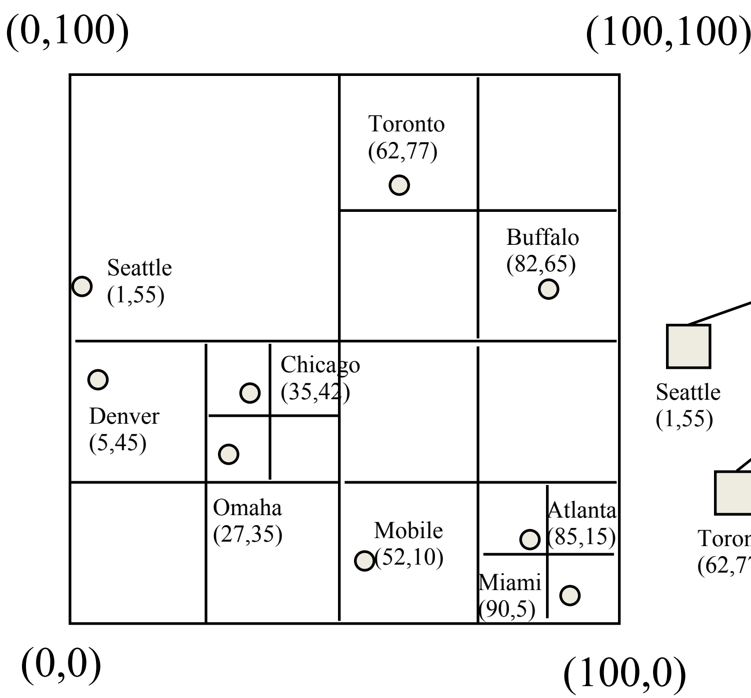
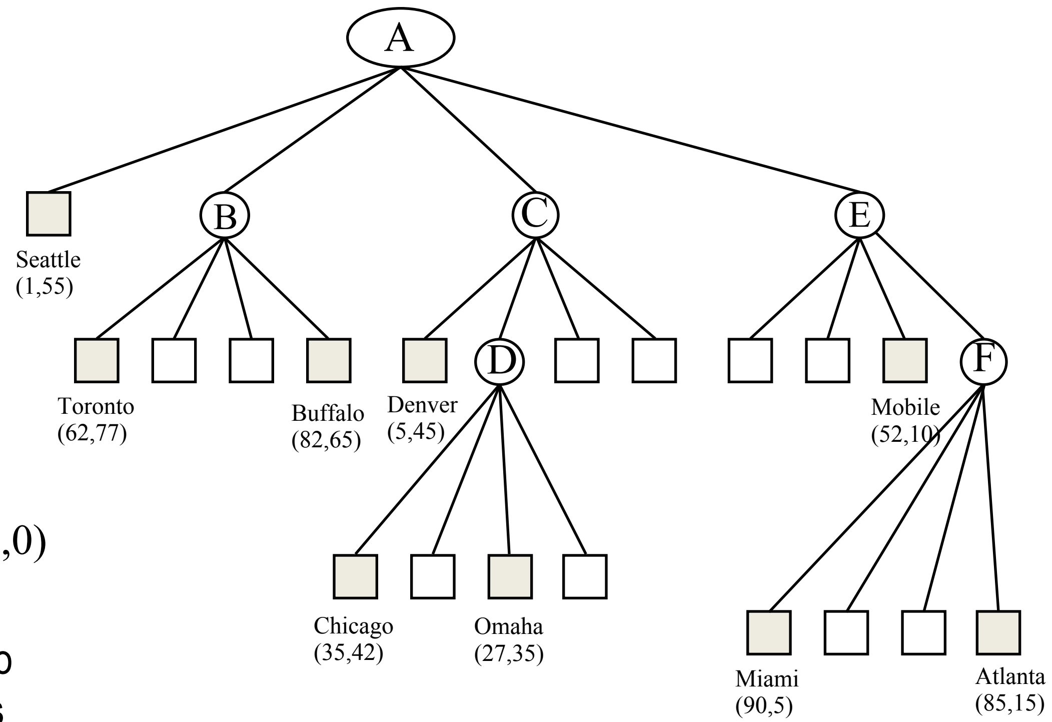
PR Quadtree

- PR (Point-Region) quadtree
- Regular decomposition (similar to Region quadtree)
- Independent of the order in which data points are inserted into it
- ☹: if two points are very close, decomposition can be very deep

Example of PR Quadtree





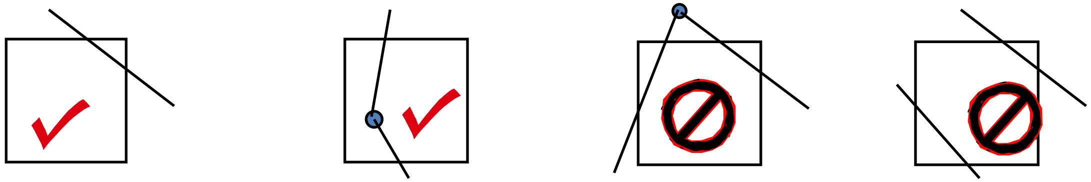
Subdivide into quadrants until the two points are located in different regions

PM Quadtree

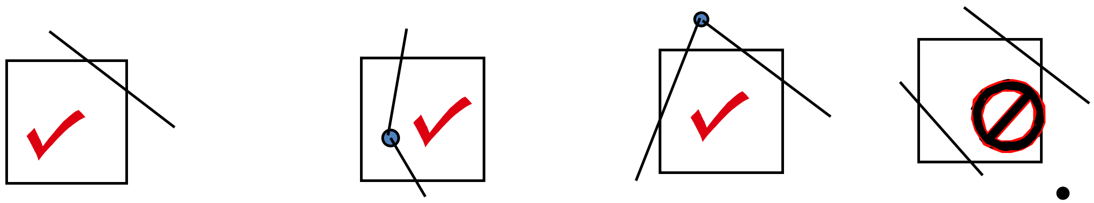
- PM (Polygonal-Map) quadtree family
  - PM1 quadtree, PM2 quadtree, PM3 quadtree, PMR quadtree, … etc.
- PM1 quadtree
  - Based on regular decomposition of space
  - Vertex-based implementation
  - Criteria
    - At most one vertex can lie in a region represented by a quadtree leaf
    - If a region contains a vertex, it can contain no partial-edge that does not include that vertex
    - If a region contains no vertices, it can contain at most one partial-edge

PM Quadtree

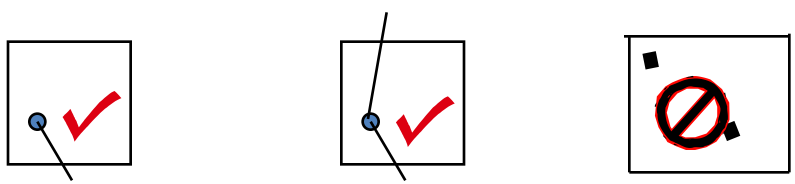
PM1 quadtree



PM2 quadtree



PM3 quadtree



- Each node in a PM quadtree is a collection of partial edges (and a vertex)
- Each point record has two field (x,y)
- Each partial edge has four field (starting_point, ending_point, left region, right region)

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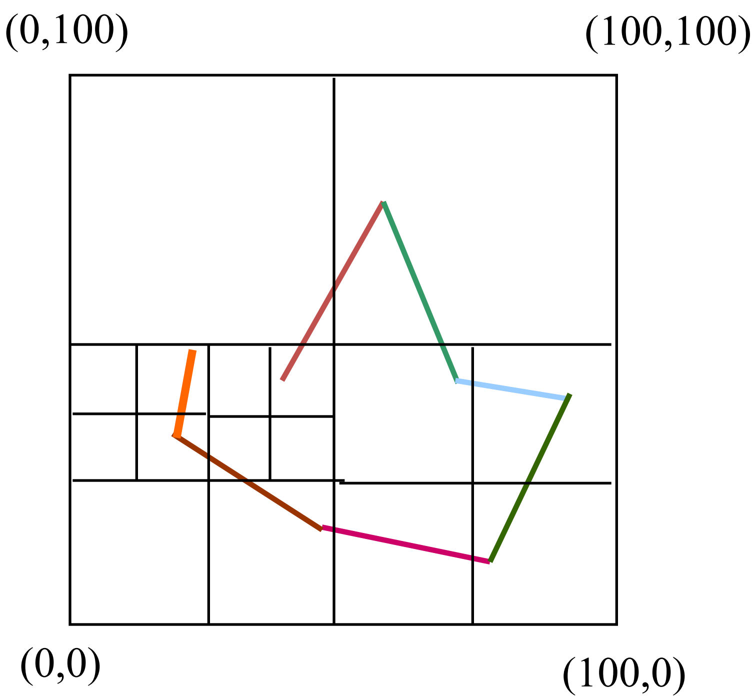
Example of PM1 Quadtree



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References

- National Technical University of Athens , Theoretical Computer Science II: Advanced Data Structures
- Jürg Nievergelt, Hans Hinterberger, Kenneth C. Sevcik: The Grid File: An Adaptable, Symmetric Multikey File Structure. ACM Trans. Database Syst. 9(1): 38-71 (1984)
- H. V. Jagadish: Linear Clustering of Objects with Multiple Atributes. ACM SIGMOD Conference 1990: 332-342