Chapter 12: Indexing and Hashing

- Basic Concepts
- Ordered Indices
- B+-Tree Index Files
- B-Tree Index Files
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- Comparison of Ordered Indexing and Hashing
- Index Definition in SQL
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Basic Concepts

- Indexing mechanisms used to speed up access to desired data.
  - E.g., author catalog in library
- **Search Key** - attribute or set of attributes used to look up records in a file.
- An **index file** consists of records (called **index entries**) of the form

search-key	pointer

- Index files are typically much smaller than the original file
- Two basic kinds of indices:
  - **Ordered indices:** search keys are stored in sorted order
  - **Hash indices:** search keys are distributed uniformly across "buckets" using a "hash function".

Index Evaluation Metrics

- Access types supported efficiently. E.g.,
  - records with a specified value in the attribute
  - or records with an attribute value falling in a specified range of values.

Indexing techniques evaluated on basis of:

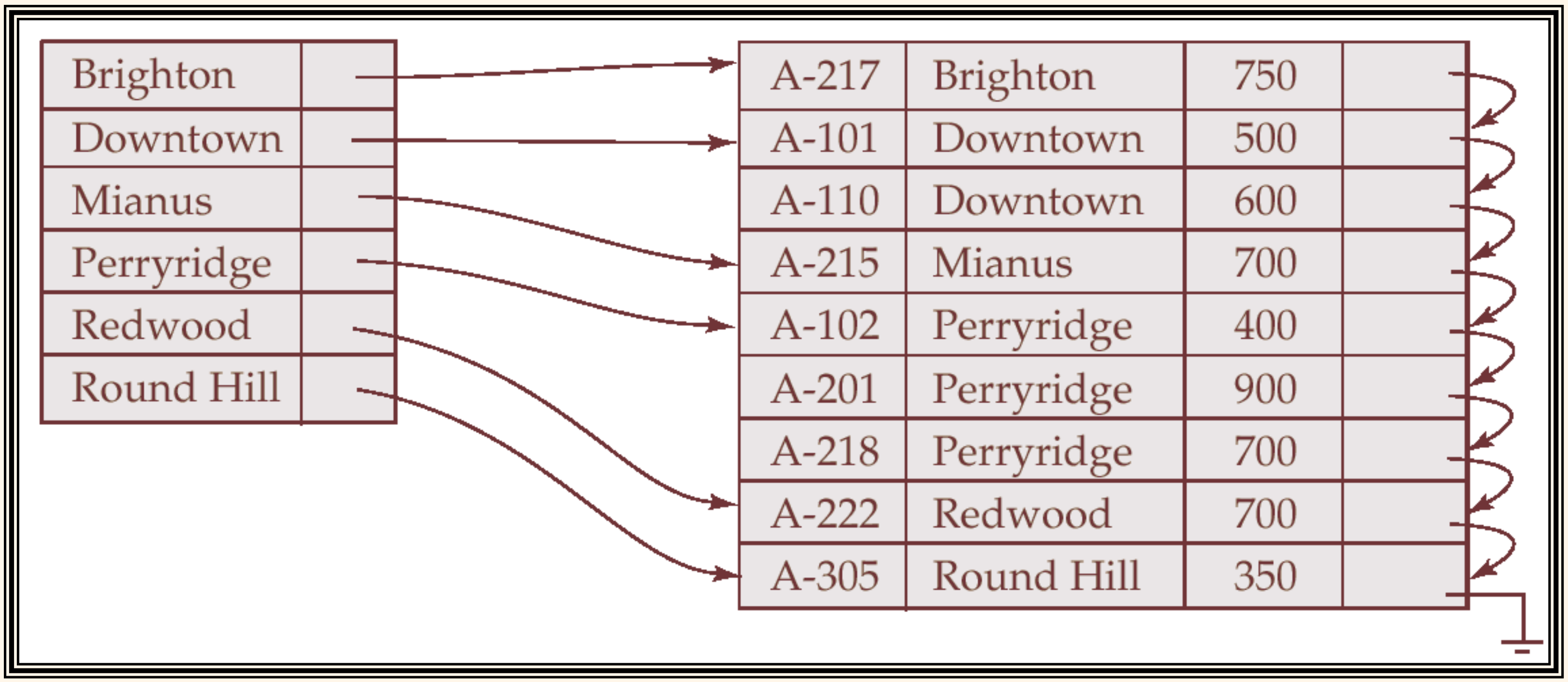
- Access time
- Insertion time
- Deletion time
- Space overhead

Ordered Indices

- In an **ordered index,** index entries are stored sorted on the search key value. E.g., author catalog in library.
- **Primary index:** in a sequentially ordered file, the index whose search key specifies the sequential order of the file.
  - Also called **clustering index /* Not! */**
  - The search key of a primary index is usually but not necessarily the primary key.
- **Secondary index**: an index whose search key specifies an order different from the sequential order of the file. Also called
  non-clustering index. **/* Wrong Again! */**
- Index-sequential file: ordered sequential file with a primary index.

Dense Index Files

- Dense index — Index record appears for every search-key value in the file.



Sparse Index Files

- Sparse Index: contains index records for only some search-key values.
  - Applicable when records are sequentially ordered on search-key
- To locate a record with search-key value *K* we:
  - Find index record with largest search-key value < *K*
  - Search file sequentially starting at the record to which the index record points
- Less space and less maintenance overhead for insertions and deletions.
- Generally slower than dense index for locating records.
- Good tradeoff: sparse index with an index entry for every block in file, corresponding to least search-key value in the block.

Example of Sparse Index Files

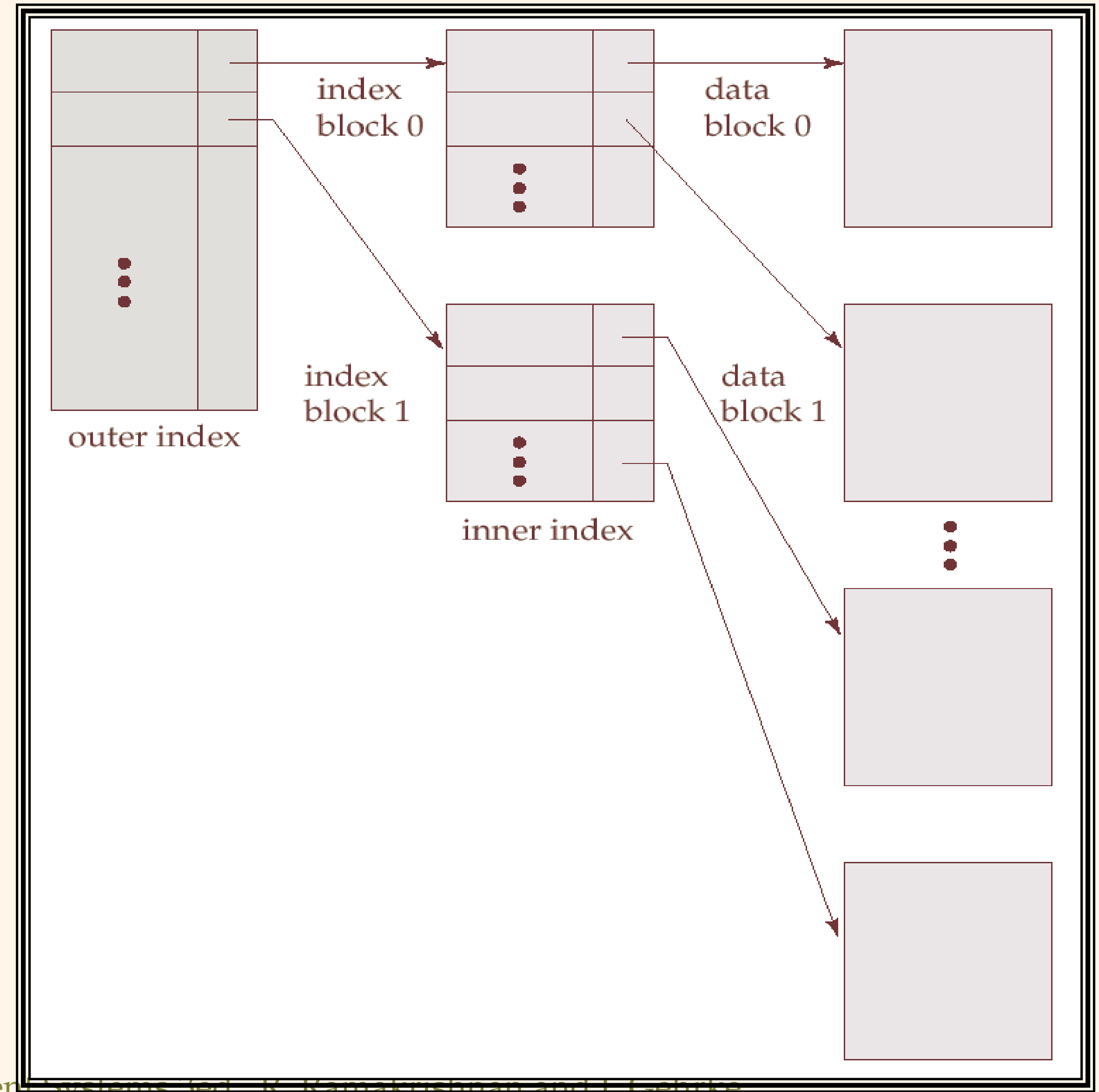
Brighton	
Mianus	
Redwood	

A-217	Brighton	750	
A-101	Downtown	500	
A-110	Downtown	600	
A-215	Mianus	700	
A-102	Perryridge	400	
A-201	Perryridge	900	
A-218	Perryridge	700	
A-222	Redwood	700	
A-305	Round Hill	350	

Multilevel Index

- If primary index does not fit in memory, access becomes expensive.
- To reduce number of disk accesses to index records, treat primary index kept on disk as a sequential file and construct a sparse index on it.
  - outer index – a sparse index of primary index
  - inner index – the primary index file
- If even outer index is too large to fit in main memory, yet another level of index can be created, and so on.
- Indices at all levels must be updated on insertion or deletion from the file.

Multilevel Index (Cont.)



Index Update: Deletion

- If deleted record was the only record in the file with its particular search-key value, the search-key is deleted from the index also.

- Single-level index deletion:
  - Dense indices – deletion of search-key is similar to file record deletion.
  - Sparse indices – if an entry for the search key exists in the index, it is deleted by replacing the entry in the index with the next search-key value in the file (in search-key order). If the next search-key value already has an index entry, the entry is deleted instead of being replaced.

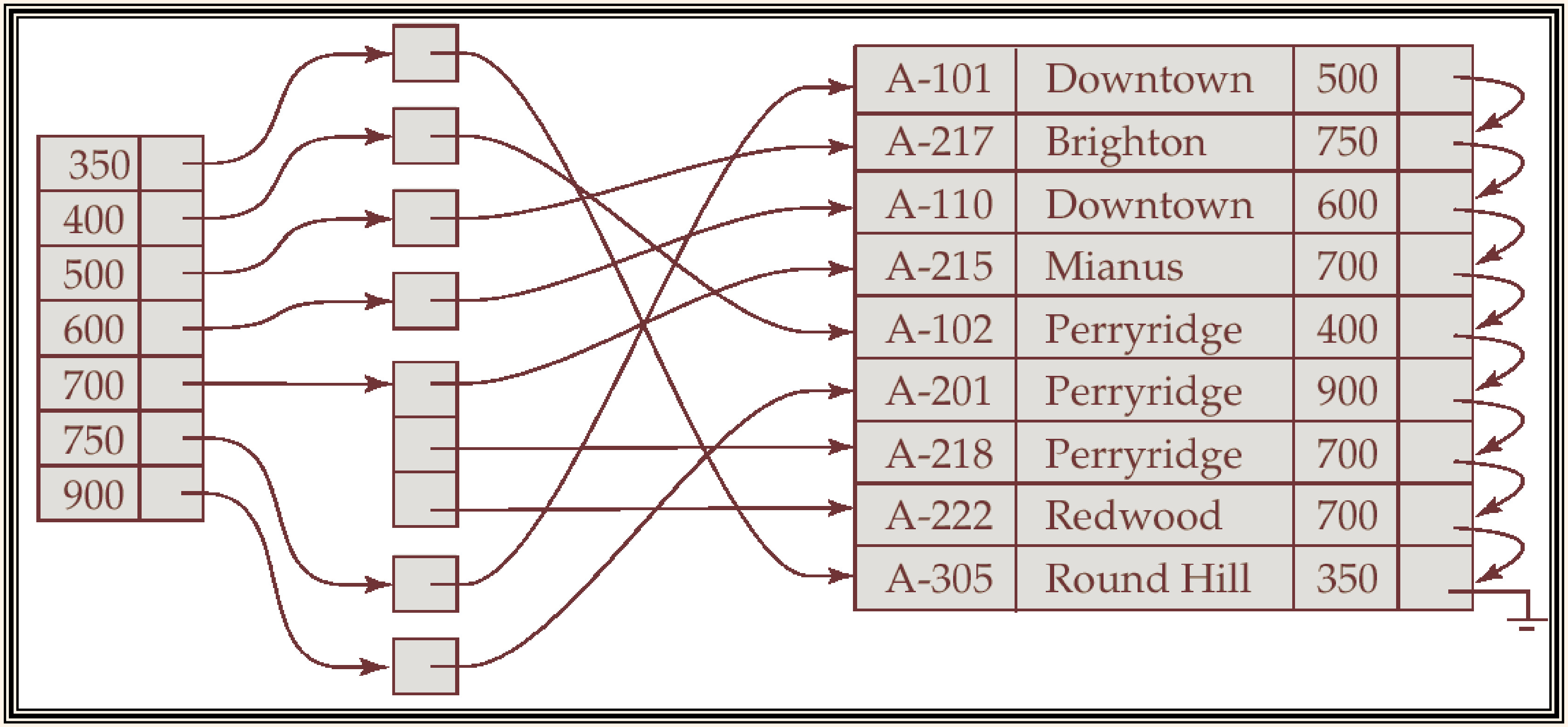
Index Update: Insertion

- Single-level index insertion:
  - Perform a lookup using the search-key value appearing in the record to be inserted.
  - Dense indices – if the search-key value does not appear in the index, insert it.
  - Sparse indices – if index stores an entry for each block of the file, no change needs to be made to the index unless a new block is created. In this case, the first search-key value appearing in the new block is inserted into the index.
- Multilevel insertion (as well as deletion) algorithms are simple extensions of the single-level algorithms

Secondary Indices

- Frequently, one wants to find all the records whose values in a certain field (which is not the search-key of the primary index satisfy some condition.
  - Example 1: In the *account* database stored sequentially by account number, we may want to find all accounts in a particular branch
  - Example 2: as above, but where we want to find all accounts with a specified balance or range of balances
- We can have a secondary index with an index record for each search-key value; index record points to a bucket that contains pointers to all the actual records with that particular search-key value.

Secondary Index on **balance** *field of* **account**



Primary and Secondary Indices

- Secondary indices have to be dense.
- Indices offer substantial benefits when searching for records.
- When a file is modified, every index on the file must be updated, Updating indices imposes overhead on database modification.
- Sequential scan using primary index is efficient, but a sequential scan using a secondary index is expensive
  - each record access may fetch a new block from disk

B^+-Tree Index Files

B^+-tree indices are an alternative to indexed-sequential files.

- Disadvantage of indexed-sequential files: performance degrades as file grows, since many overflow blocks get created. Periodic reorganization of entire file is required.
- Advantage of B^+-tree index files: automatically reorganizes itself with small, local, changes, in the face of insertions and deletions. Reorganization of entire file is not required to maintain performance.
- Disadvantage of B^+-trees: extra insertion and deletion overhead, space overhead.
- Advantages of B^+-trees outweigh disadvantages, and they are used extensively.

B^+-Tree Index Files (Cont.)

A B^+-tree is a rooted tree satisfying the following properties:

- All paths from root to leaf are of the same length
- Each node that is not a root or a leaf has between $\lceil n/2 \rceil$ and n children.
- A leaf node has between $\lceil (n-1)/2 \rceil$ and $n-1$ values
- Special cases:
  - If the root is not a leaf, it has at least 2 children.
  - If the root is a leaf (that is, there are no other nodes in the tree), it can have between 0 and (n–1) values.

B^+-Tree Node Structure

- Typical node

P_1	K_1	P_2	...	P_{n-1}	K_{n-1}	P_n

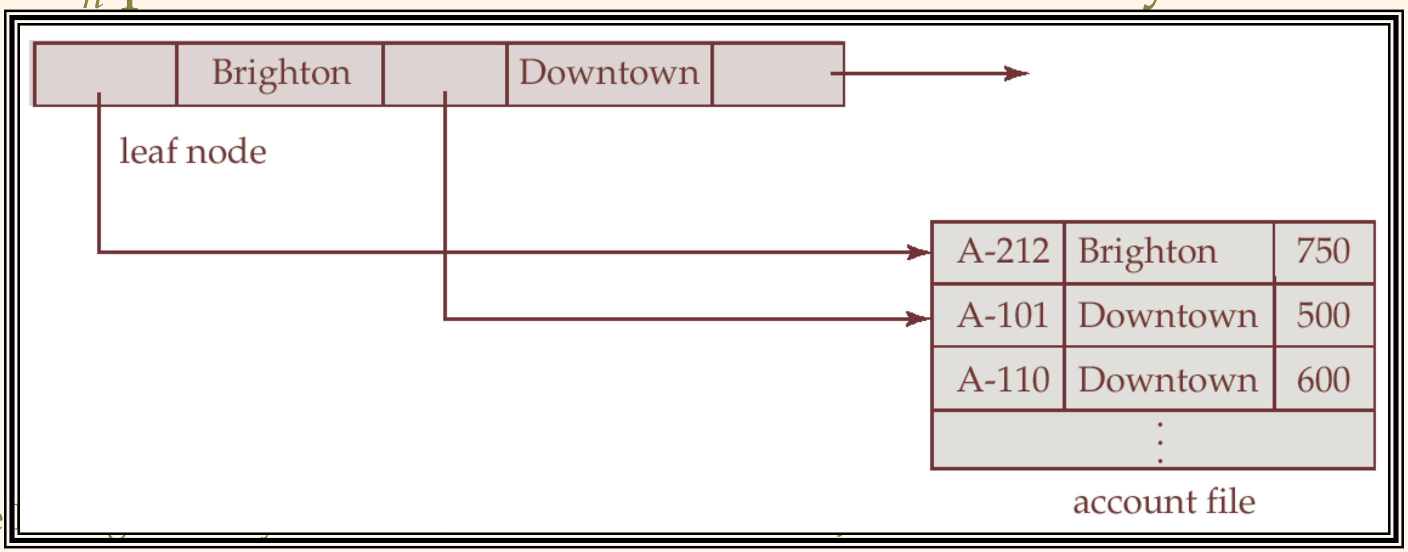
  - K_i are the search-key values
  - P_i are pointers to children (for non-leaf nodes) or pointers to records or buckets of records (for leaf nodes).
- The search-keys in a node are ordered

$$K_1 < K_2 < K_3 < \ldots < K_{n-1}$$

Leaf Nodes in B^+-Trees

Properties of a leaf node:

- For $i = 1, 2, \ldots, n-1$, pointer P_i either points to a file record with search-key value K_i, or to a bucket of pointers to file records, each record having search-key value K_i. Only need bucket structure if search-key does not form a primary key. /* if not primary index! */
- If L_i, L_j are leaf nodes and $i < j$, L_i's search-key values are less than L_j's search-key values
- P_n points to next leaf node in search-key order

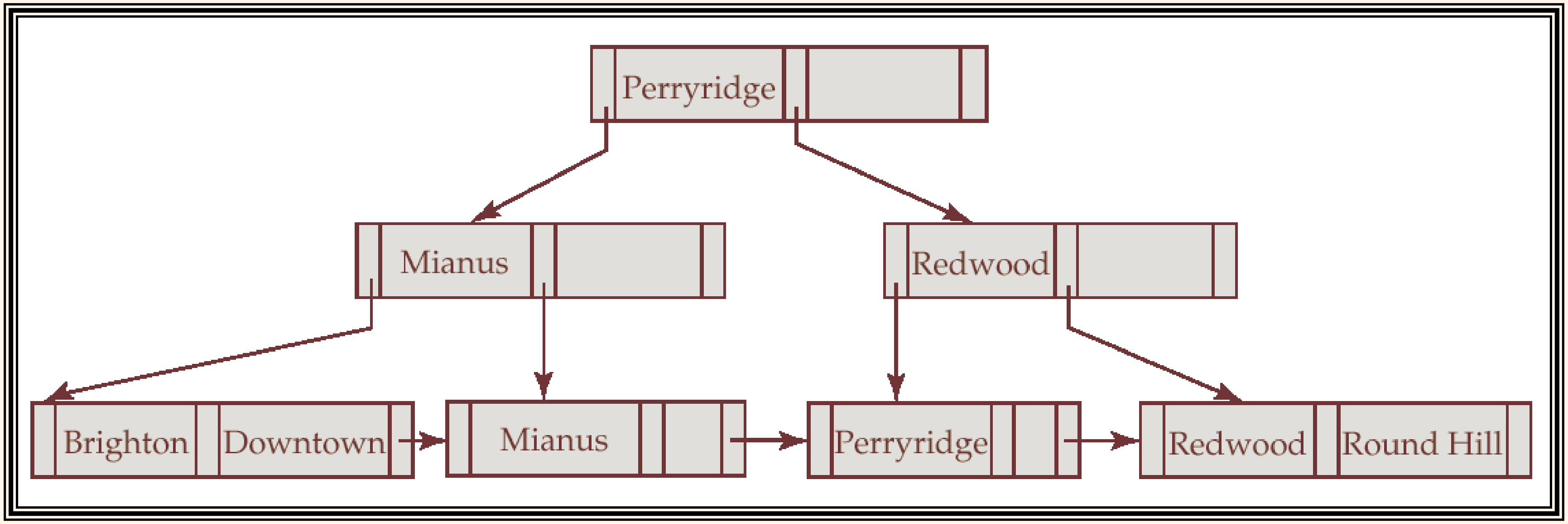


Non-Leaf Nodes in B+-Trees

- Non leaf nodes form a multi-level sparse index on the leaf nodes. For a non-leaf node with *m* pointers:
  - All the search-keys in the subtree to which P_1 points are less than K_1
  - For $2 \leq i \leq n - 1$ /* n */, all the search-keys in the subtree to which P_i points have values greater than or equal to K_{i-1} and less than K_{m-1} /* K_i except for i=n */,

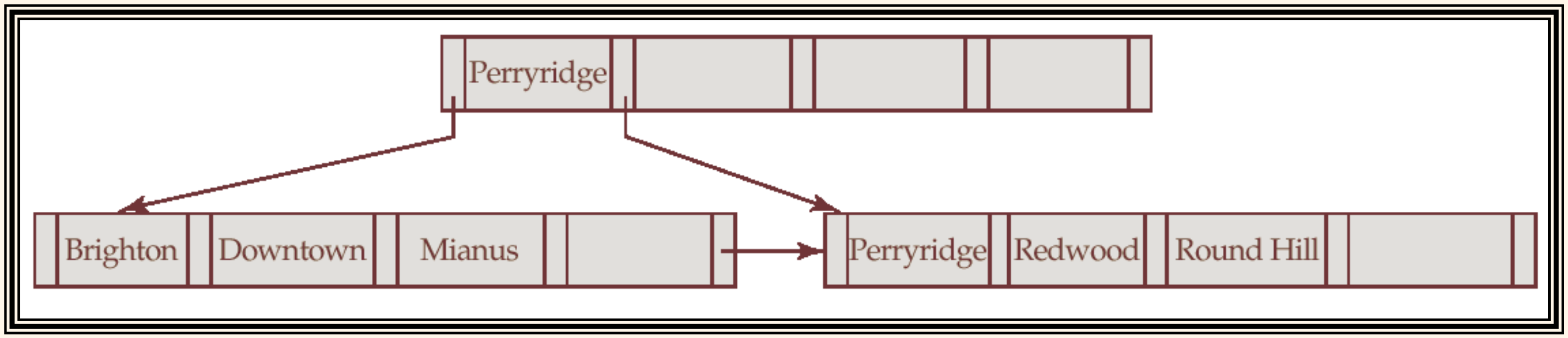
P_1	K_1	P_2	. . .	P_{n-1}	K_{n-1}	P_n

Example of a B+-tree



B+-tree for *account* file ($n = 3$)

Example of B+-tree



B+-tree for *account* file ($n = 5$)

- Leaf nodes must have between 2 and 4 values ($\lceil (n-1)/2 \rceil$ and $n-1$, with $n = 5$).
- Non-leaf nodes other than root must have between 3 and 5 children ($\lceil (n/2) \rceil$ and n with n =5).
- Root must have at least 2 children.

Observations about B^+-trees

- Since the inter-node connections are done by pointers, "logically" close blocks need not be "physically" close.
- The non-leaf levels of the B^+-tree form a hierarchy of sparse indices.
- The B^+-tree contains a relatively small number of levels (logarithmic in the size of the main file), thus searches can be conducted efficiently.
- Insertions and deletions to the main file can be handled efficiently, as the index can be restructured in logarithmic time (as we shall see).